Solutions

GW 2.19

Let's assume we have a 3x3 neighborhood median filter, positioned at some position p.

Image $A = \begin{bmatrix} a & a & a \\ a & a & b \\ b & b & b \end{bmatrix}$ with $a \neq b$

Image $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 0 \end{bmatrix}$ with $c = b - a$

Median of A is "a"
Median of B is "0"

$$A + B = \begin{bmatrix} a & a & a \\ a & b & b \\ b & b & b \end{bmatrix}$$

median of A + B is "b".

Since median A + median B $\neq$ median (A + B), the median operator is <u>not</u> linear.

3.10

Given $P_r(r)$, transform to new $P_z(z)$. follow method from Gw 3.3.2.

$$T(r) = \int_0^r P_r(w)\,dw = \int_0^r (2-2w)\,dw$$

$$= 2w - \frac{2w^2}{2}\Big|_0^r = 2r - r^2$$

$$G(z) = \int_0^z P_z(t)\,dt = \int_0^z 2t\,dt$$

$$= \frac{2t^2}{2}\Big|_0^z = z^2$$

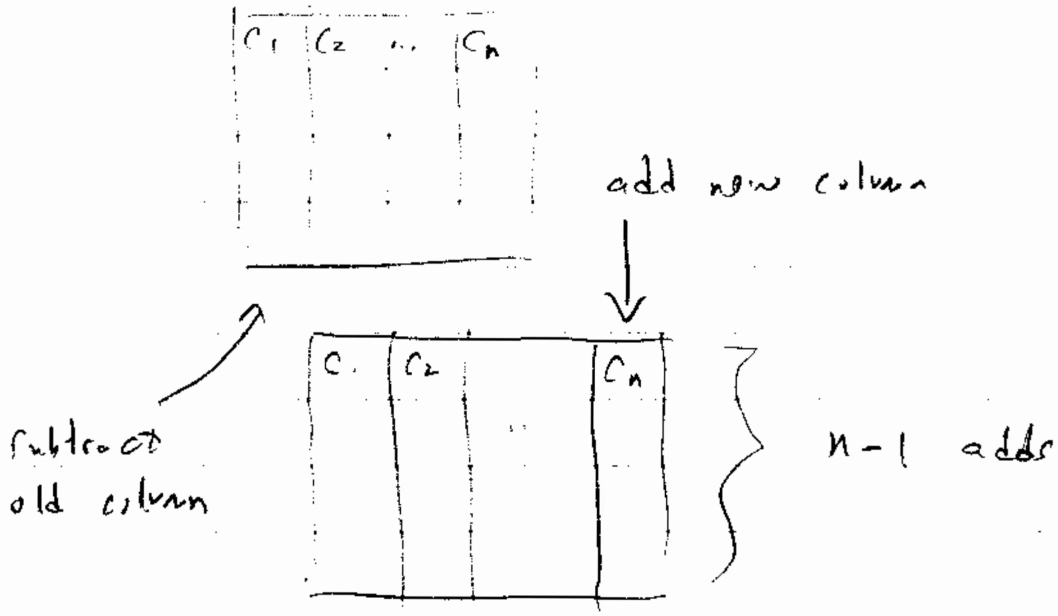
So, from equation 3.3-12, the transformation is

$$z = G^{-1}(T(r)) = \sqrt{2r - r^2}$$

3.17 Box filter vs. Brute force

Brute force requires $n \times n - 1$ adds/pixel, for a total of $(n^2-1)\cdot N \cdot M$ operations.

Box filter requires n^2-1 adds for first location, but only $n+1$ adds for each location after that



Total is $\underbrace{(n^2-1)}_{\text{ignore}} + (n+1)(N\cdot M - 1)$

Computational advantage is $\dfrac{n^2-1}{n+1} = \boxed{n-1}$

4

$$\text{Mask} = \frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix} \rightarrow y$$

$$\downarrow x$$

$$H(\Omega_1, \Omega_2) = \sum_x \sum_y h(x,y)\, e^{-j(\Omega_1 x + \Omega_2 y)}$$

$$= \Big(4 + 2e^{-j\Omega_2} + 2e^{+j\Omega_2} \qquad \left(\begin{matrix}\text{middle}\\ \text{row}\end{matrix}\right)$$

$$+ 2e^{-j\Omega_1} + e^{+j\Omega_1} \qquad \left(\begin{matrix}\text{middle}\\ \text{column}\end{matrix}\right)$$

$$+ e^{-j\Omega_1} e^{-j\Omega_2} + e^{-j\Omega_1} e^{+j\Omega_2} +$$

$$e^{+j\Omega_1} e^{-j\Omega_2} + e^{+j\Omega_1} e^{+j\Omega_2} \Big) / 16$$

$$= \frac{1}{16}\Big[4 + 4\cos\Omega_1 + 4\cos\Omega_2 +$$

$$4\cos\Omega_1 \cos\Omega_2 \Big]$$

(5)

b) for $\Omega_2 = 0$,

$$H(\Omega_1, 0) = \frac{1}{16}\left[4 + 4\cos\Omega_1 + 4 + 4\cos\Omega_1\right]$$

$$= \frac{1}{16}\left[8 + 8\cos\Omega_1\right]$$

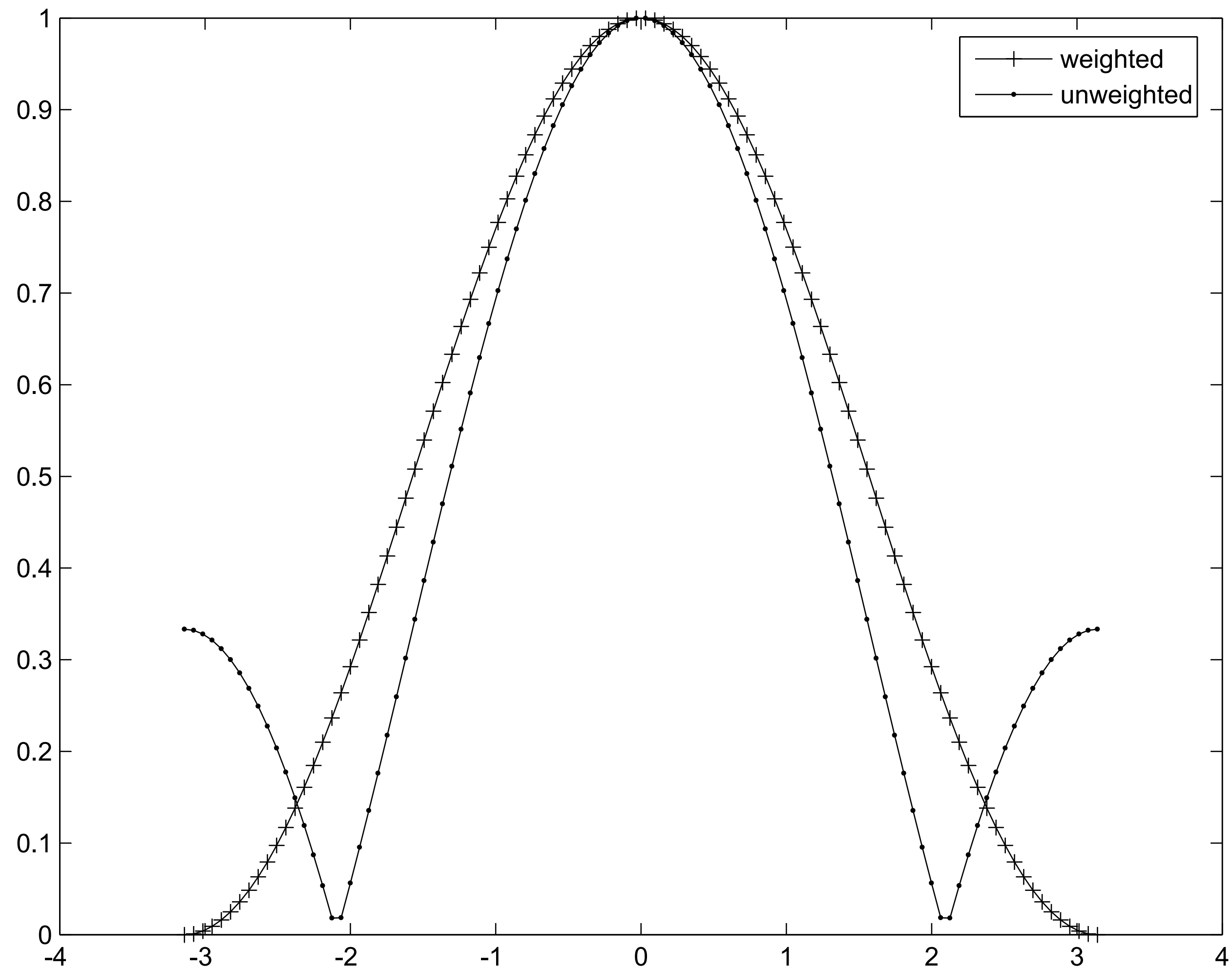
$$= \frac{1}{2}\left[1 + \cos\Omega_1\right]$$

Compare this to 3×3 averaging mask,

$$H(\Omega_1) = \frac{1}{3}(1 + 2\cos\Omega_1)$$

The weighted mask has broader passband, but better high freq. attenuation.

See attached plot.


weighted
unweighted
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4