Solutions

4.14

$$g(x,y) = \frac{1}{4}\left[f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1) \right]$$

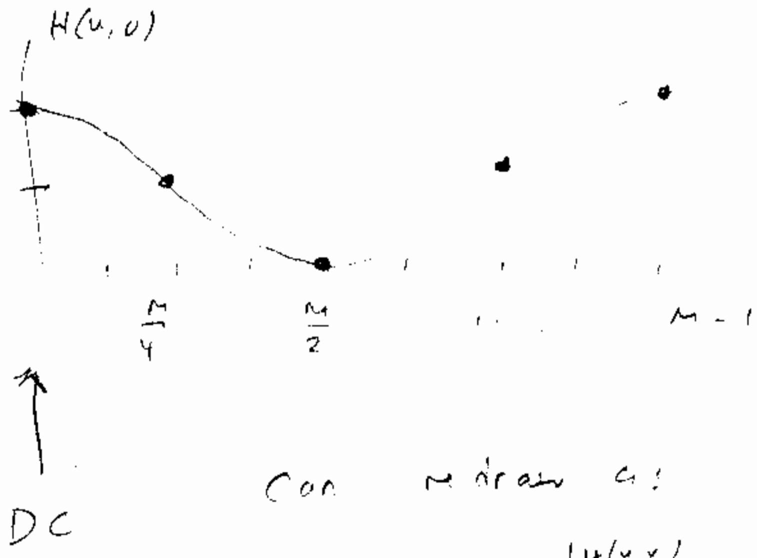
a)

$$H(u,v) = \frac{1}{4}\left[e^{-j2\pi u/M} + e^{j2\pi u/M} + e^{-j2\pi v/N} + e^{j2\pi v/N} \right]$$

$$= \frac{1}{4}\left[2\cos\left(\frac{2\pi u}{M}\right) + 2\cos\left(\frac{2\pi v}{N}\right) \right]$$

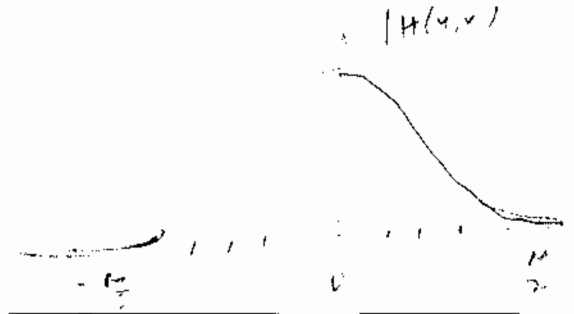
$$= \frac{1}{2}\cos\left(\frac{2\pi u}{M}\right) + \frac{1}{2}\cos\left(\frac{2\pi v}{N}\right)$$

b) Consider $H(u,0)$, $u = 0, 1, \ldots, M-1$



LPF

Can redraw as:



4.15

$$g(x,y) = f(x+1, y) - f(x,y)$$

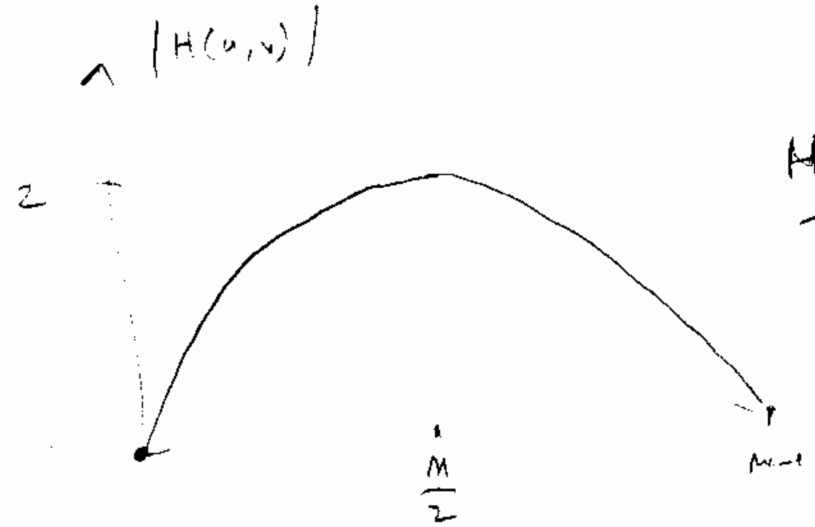
a) $$H(u,v) = e^{+j 2\pi u/M} - 1$$

$$= e^{j 2\pi u/2M} \left(e^{j 2\pi u/2M} - e^{-j 2\pi u/2M} \right)$$

$$= e^{j 2\pi u/2M} \cdot 2j \sin\left(\frac{2\pi u}{2M}\right)$$

$$= 2j\, e^{j \pi u/M} \sin\left(\frac{\pi u}{M}\right)$$

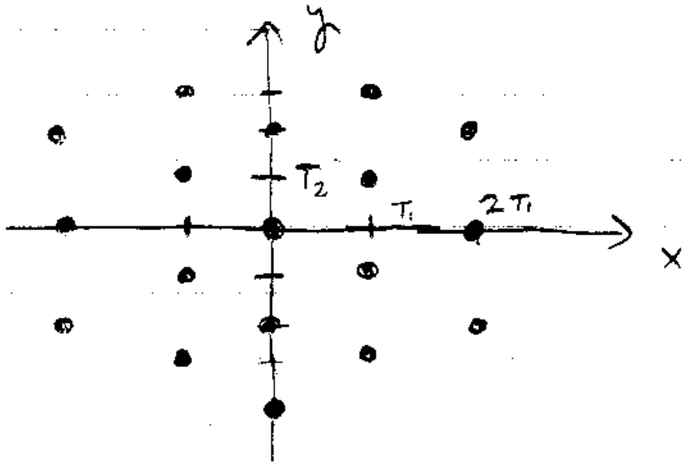
b) Magnitude response is $|H(u,v)| = \left| 2 \sin \frac{\pi u}{M} \right|$



HPF

Problem 3)

Assume a regular hexagonal sampling grid.



T_1 Sampling periods
T_2

$$T_2 = \frac{\sqrt{3}}{3} T_1$$

a)

Can write $s(x,y)$ as the sum of δ-functions

$$s(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - 2nT_1,\ y - 2mT_2)$$

$$+ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - 2nT_1 - T_1,\ y - 2mT_2 - T_2)$$

b)

If $f(x,y) = \sum_{n} \sum_{m} \delta(x - nT_1,\ y - mT_2)$,

$$F(u,v) = \int_{-\infty}^{\infty}\int_{\infty}^{\infty} f(x,y)\, e^{-j(\Omega_1 x + \Omega_2 y)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty} \sum_{n} \delta(x - nT_1)\, e^{-j\Omega_1 x}\, dx \cdot$$

$$\int_{-\infty}^{\infty} \sum_{m} \delta(y - nT_2)\, e^{-j\Omega_2 y}\, dy$$

$$= \frac{2\pi}{T_1} \sum_{n} \delta\left(\Omega_1 - \frac{2\pi n}{T_1}\right) \cdot \frac{2\pi}{T_2} \sum_{m} \delta\left(\Omega_2 - \frac{2\pi n}{T_2}\right)$$

$$= \frac{(2\pi)^2}{T_1 T_2} \sum_{n} \sum_{m} \delta\left(\Omega_1 - \frac{2\pi n}{T_1},\ \Omega_2 - \frac{2\pi m}{T_2}\right)$$

So, for our $s(x,y)$,

$$S(\Omega_1,\Omega_2) = \frac{(2\pi)^2}{2T_1 \cdot 2T_2} \sum_n \sum_m \delta\left(\Omega_1 - \frac{2\pi n}{2T_1}, \Omega_2 - \frac{2\pi m}{2T_2}\right)$$

$$+ \frac{(2\pi)^2}{2T_1 \cdot 2T_2} e^{-j(\Omega_1 T_1 + \Omega_2 T_2)} \sum_m \sum_n \delta\left(\Omega_1 - \frac{2\pi n}{2T_1}, \Omega_2 - \frac{2\pi m}{2T_2}\right)$$

$$= \frac{(2\pi)^2}{4T_1T_2} \sum_m \sum_n \left(1 + e^{-j(\Omega_1 T_1 + \Omega_2 T_2)}\right) \cdot \delta\left(\Omega_1 - \frac{\pi n}{T_1}, \Omega_2 - \frac{\pi m}{T_2}\right)$$

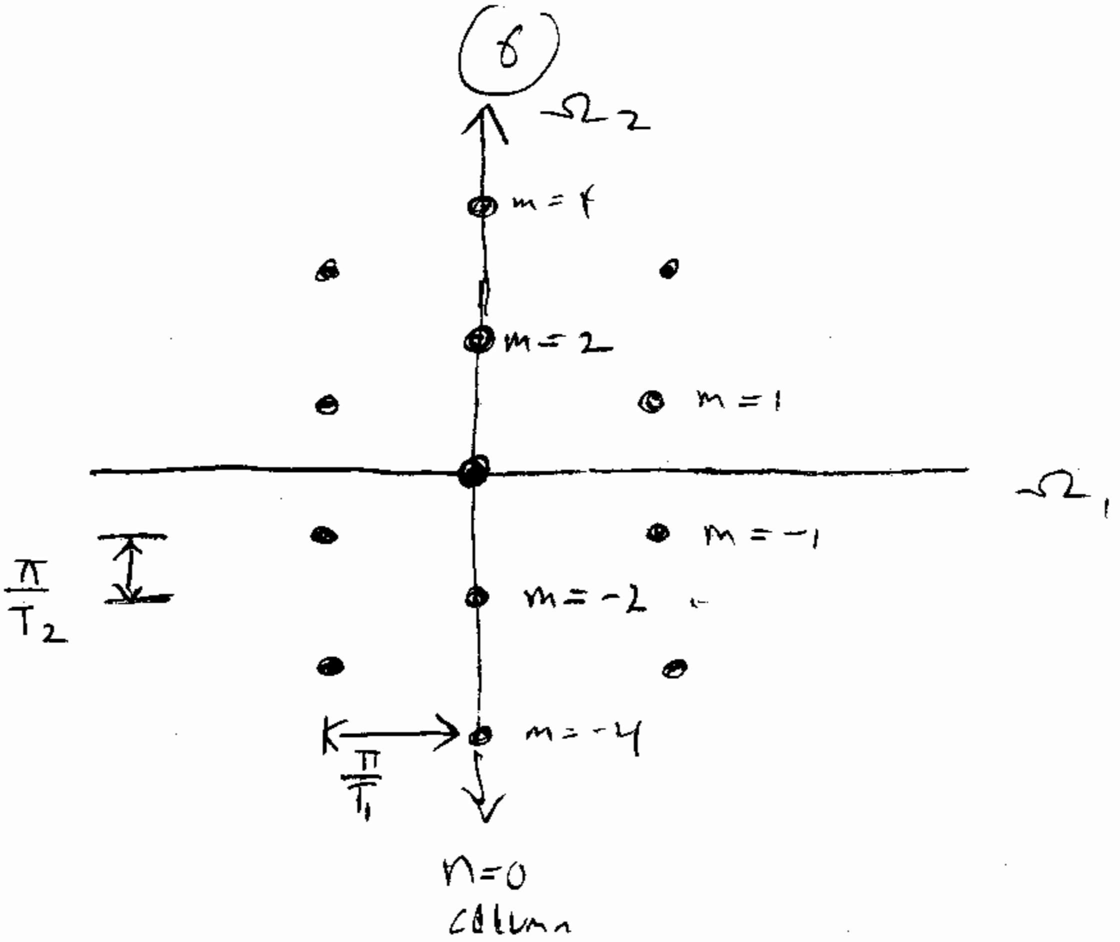
The term $1 + e^{-j(\Omega_1 T_1 + \Omega_2 T_2)}$ is zero for $m+n$ odd and is equal to two for $m+n$ even.

So

$$S(\Omega_1,\Omega_2) = \frac{2\pi^2}{T_1T_2} \sum_{\substack{m \\ m+n \text{ even}}} \sum_n \delta\left(\Omega_1 - \frac{\pi n}{T_1}, \Omega_2 - \frac{\pi m}{T_2}\right)$$

whew!

(6)

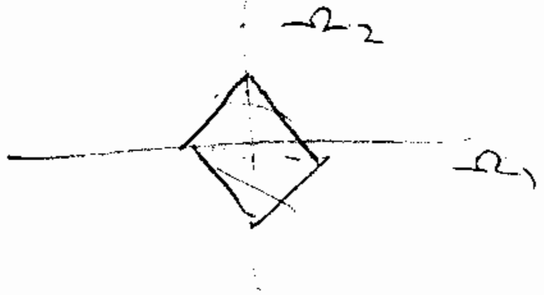


Result: Sampling function has FT that is also hexagonal.

Since $g(x,y) = f(x,y) \cdot s(x,y)$

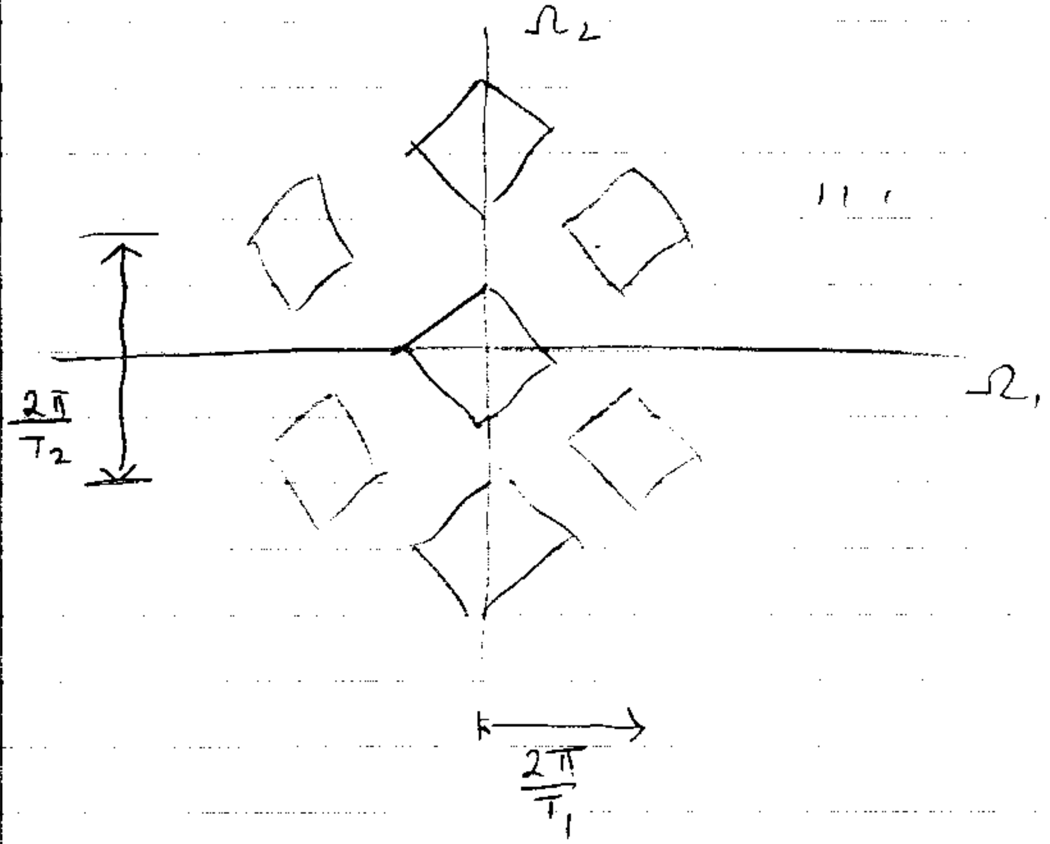
$$G(\Omega_1, \Omega_2) = F(\Omega_1, \Omega_2) * S(\Omega_1, \Omega_2)$$

If $G(\Omega_1, \Omega_2)$ looks like this:



7

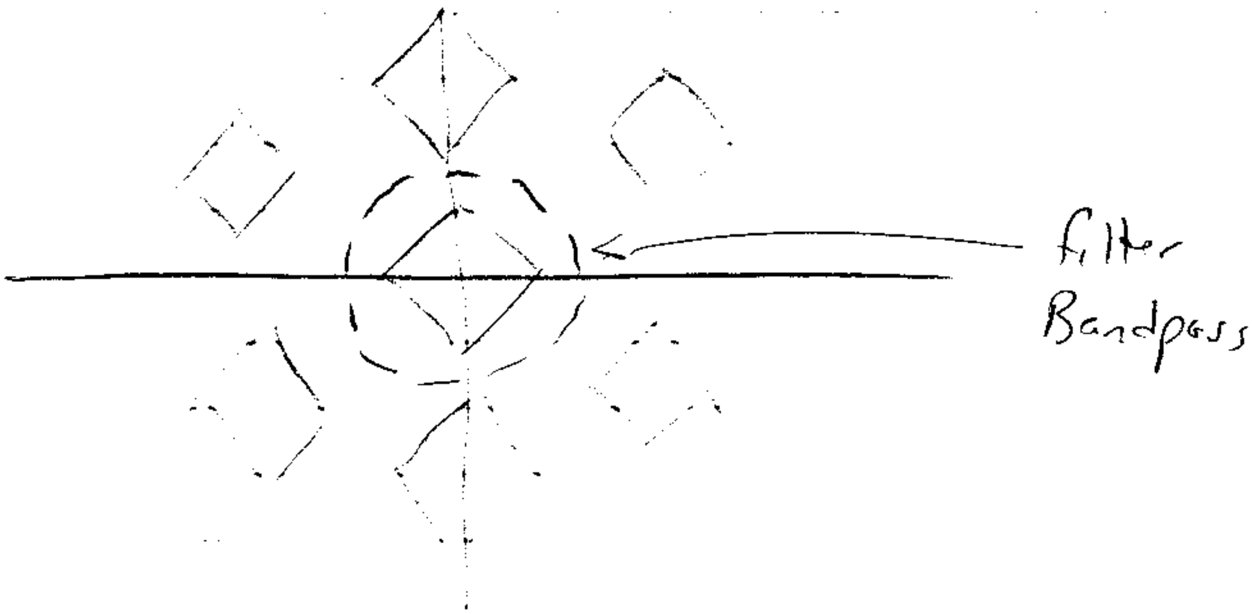
Then $G(\Omega_1, \Omega_2)$ is



So,

$$G(\Omega_1,\Omega_2) = \frac{2\pi^2}{T_1 T_2} \sum_{\substack{m \\ m+n \text{ even}}} \sum_{n} F\left(\Omega_1 - \frac{\pi n}{T_1}, \Omega_2 - \frac{\pi m}{T_2}\right)$$

c) Design filter:



Choose D_0 & n for best image quality, with minimum blur.