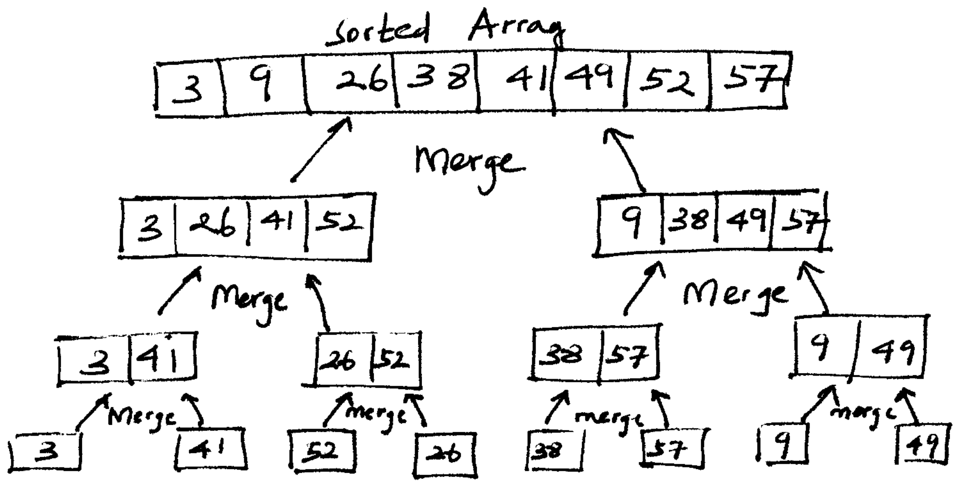
SOLUTIONS FOR HW #1

055:133

1.



2.

$f(n) = O(g(n))$

$\Rightarrow$ $f(n) \leq c g(n)$ for $n \geq n_0$ and any positive integer c.

$\leq c' k g(n)$ where $c = c'k$ $c' > 0$ and $k > 0$

$\leq c'[k * g(n)]$

$\therefore$ $f(n) \leq c'[k * g(n)]$ for $c' > 0$ and $n \geq n_0$

$\Rightarrow$ $f(n) = O(k \cdot g(n))$ Q.E.D

3.

① $\Rightarrow f_1(n) = O(g_1(n)) \Rightarrow f_1(n) \leq c_1 g_1(n)$ for $c_1 > 0$ and $n \geq n_{01}$

② $\Rightarrow f_2(n) = O(g_2(n)) \Rightarrow f_2(n) \leq c_2 g_2(n)$ for $c_2 > 0$ and $n \geq n_{02}$

① * ② $f_1(n) * f_2(n) \leq c_1 c_2 \; g_1(n) * g_2(n)$ for $n_0 > \max\{n_{01}, n_{02}\}$

$\leq c \; g_1(n) * g_2(n)$ for $c = c_1 c_2 > 0$

$\therefore$ $f_1(n) * f_2(n) = O(g_1(n) * g_2(n))$ Q.E.D.

3

2.1 a) $P(n) = O(n^k)$ iff $k \geq d$

We have to prove $P(n) \leq cn^k$ for some $n \geq n_0$

i.e, $a_0 + a_1 n + \cdots + a_d n^d \leq cn^k$ for $n \geq n_0$

$\Rightarrow a_0 n^{-k} + a_1 n^{-(k-1)} + \cdots a_d n^{-(k-d)} \leq c$ for $n \geq n_0$

But we know that $k \geq d$ (given)

$\therefore$ All the terms in LHS have $n^{(k-i)}$ in the denominator

and if we choose $n \geq 1$ (ie, $n_0 = 1$)

then c can be choosen as

$$c = \sum_{i=0}^{d} a_i \quad \text{such that}$$

$$a_0 n^{-k} + a_1 n^{-(k-1)} + \cdots a_d n^{-(k-d)} \leq \sum_{i=0}^{d} a_i$$

$\Rightarrow$ for $c = \sum_{i=0}^{d} a_i$ $\quad n \geq n_0 \; (>1)$

we have $P(n) \leq cn^k$ for $k \geq d$

Thus proved.

4

2.4 a) $f(n) = O(g(n)) \Rightarrow f(n) \leq cg(n)$ for $n \geq n_0$ —— ①

If $g(n) = O(f(n))$ by transpose symmetry

$f(n) = \Omega(g(n))$

$\Rightarrow 0 \leq cg(n) \leq f(n)$ for $n \geq n_0'$

But this is contradictory to ①

Hence It is disproved.

Dis Proof by example : Take $f(n) = n$ $\quad g(n) = n^2$

$0 \leq n \leq cn^2$ for all $n \geq n_0$ $\quad n \geq 1, \; c = 1$

$\therefore f(n) = O(g(n))$

Checking for $g(n) = O(f(n))$

$0 \leq n^2 \overset{?}{\leq} cn \quad \forall\ n \geq n_0$

$0 \leq n \leq c \quad \forall\ (n \geq n_0)$

No! for some large n_0

$g(n) \neq O(f(n))$.

⑤ 2.4 d) $f(n) = O(g(n)) \Rightarrow f(n) \leq c\,g(n)$ for $c > 0$ $n \geq n_0$

We have to check if $2^{f(n)} = O(2^{g(n)})$

or, if $2^{f(n)} \leq c' 2^{g(n)}$ for $n \geq n_0'$

$\Rightarrow \quad f(n) \log 2 \leq \log c' + g(n) \log 2$

$f(n) \leq \left[\dfrac{\log c'}{\log 2}\right] + g(n)$

$f(n) \leq K + g(n)$ a linear relationship.

This is not true always

Say $f(n) = 2n$ $g(n) = n$ $c = 5$

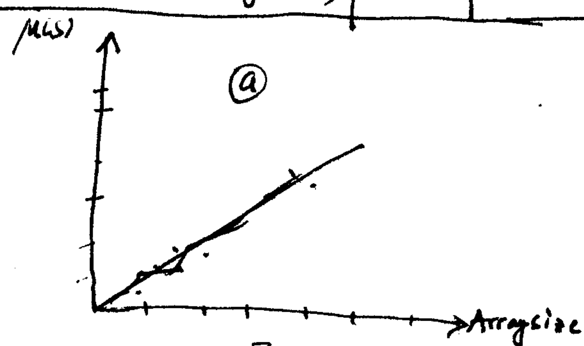
$2n \leq n + K$ won't hold true as n grows.

Thus disproved.

⑥ A typical Example of an Answer should be:

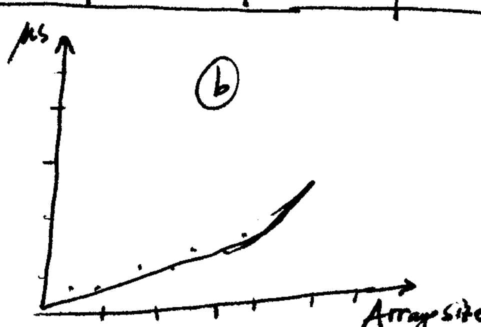
ARRAY SIZE

	1000	2000	3000	4000	8000	9000	10000
Sorted (increasing order)	0.1 μs	0.15	0.2	0.3	0.5	0.5	0.6
Random Order	10.2	14.1	23.1	32.4	61.8	77.9	90.1
Unsorted (decreasing order)	14.8	22.7	31.6	45.2	89.1	110.2	156.1



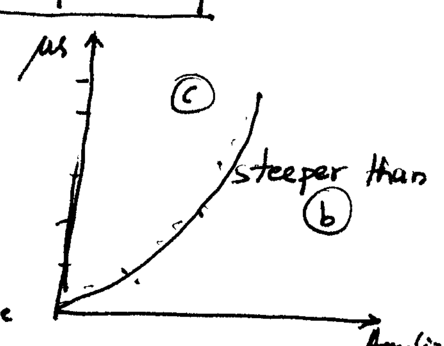
Increasing Order

linear



Random

$O(n^2)$



Decreasing Order

$O(n^2)$