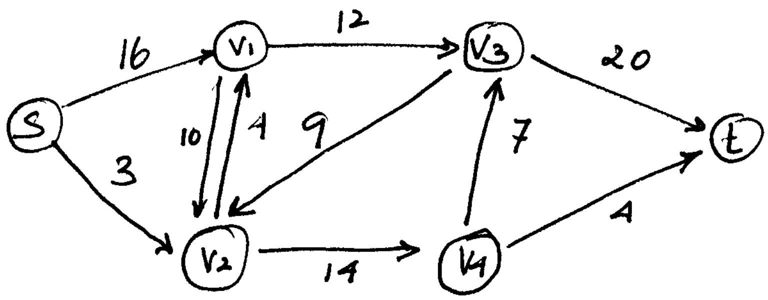
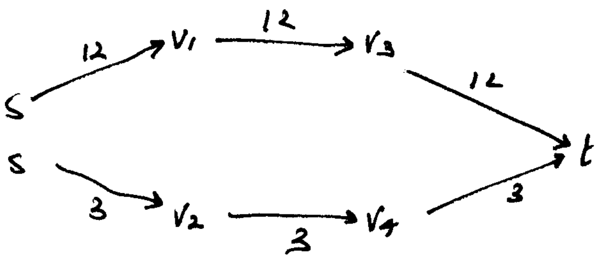
HW #5
SOLUTIONS

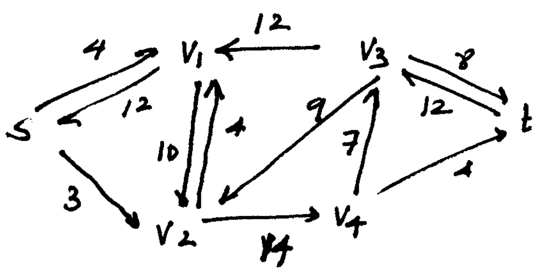
1

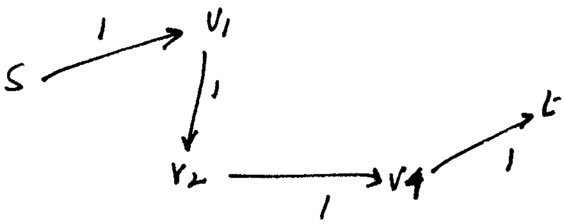


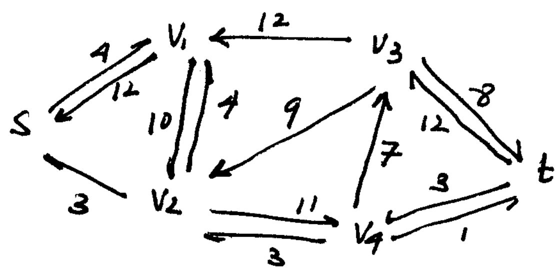
Augmented Path

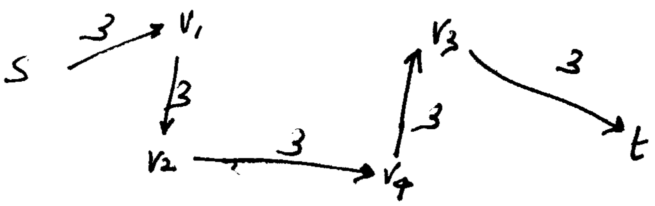
Residual Network

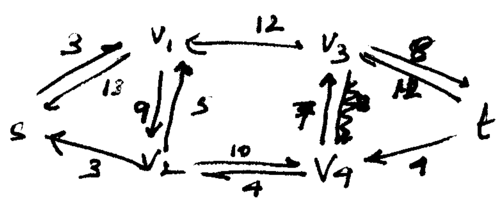


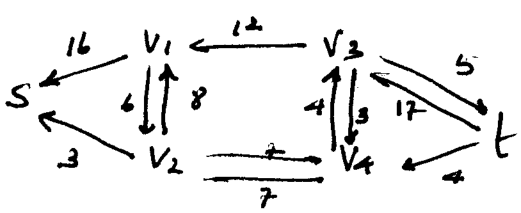




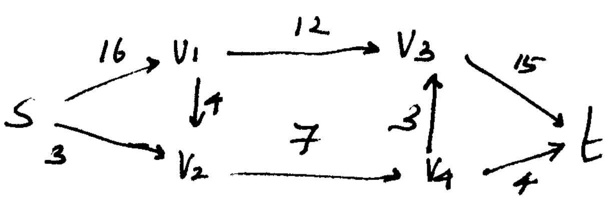




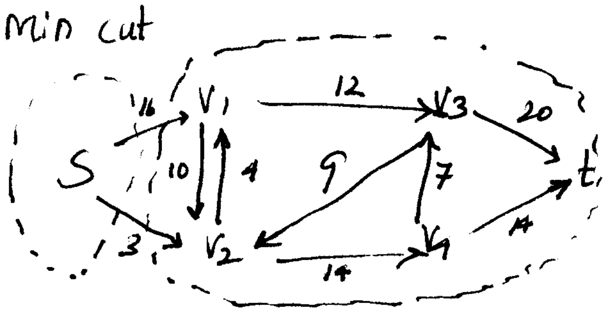




max flow = 12 + 3 + 1 + 3 = 19



Min cut



Q2: lets consider the paths after finding maximum flow.

For a $G'(v,e)$, the edges have the capacity of flow they would have in the case of max flow.

Now set Augmented path $A = \Phi$;

Find a path from s to t and mark it as p. The path has the capacity of the smallest capacity edge has, that is $f = c(e_i)$

set $A = A + P$;

Remove this flow from the graph. That is to reduce the capacity of the ~~smallest capacity edge has~~, ~~that is~~ edge that is in p by the flow of p. By doing this we are effectively removing e_i from the graph.

Now, repeat this procedure until we have no more path from s to t, which denotes that the augmented path in A will add up to the max flow.

So each time we do this we remove 1 edge and hence we can repeat only $|E|$ times.

∴ Almost there can only be $|E|$ augmented paths. QED.

Q3: a) Replace each vertex with 2 vertices V', V'' with an edge (V', V'') between them. Now all edges that enter V' will also enter V'' and all those leave V'' will also leave V' in effect.

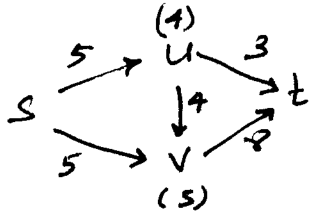
Now apply Ford-Fulkerson's max flow algorithm to the new graph. The max flow obtained now will sure satisfy both edge and vertex capacity constraints.

Proof: Edge capacity constraints is easily seen. For vertex capacity constraint, flow through v equals the flow through V' and V''

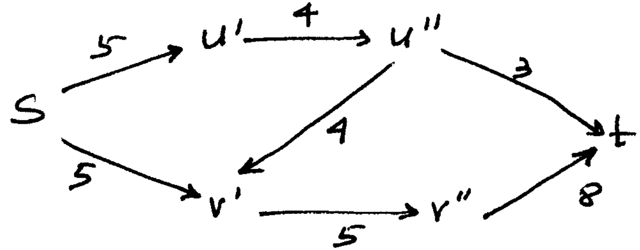
Since V' only one out edge and V'' one in edge (ie, (V', V''))

flow through (V', V'') equals the flow through V' and V'' respectively. ∵ $C(V', V'') = C(V)$ the flow is smaller than $C(V)$ due to edge capacity constraint. QED!

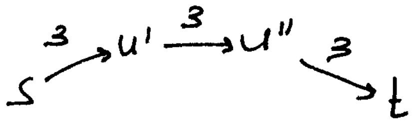
b)

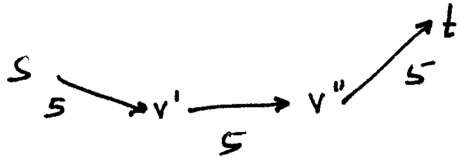


$\Rightarrow$

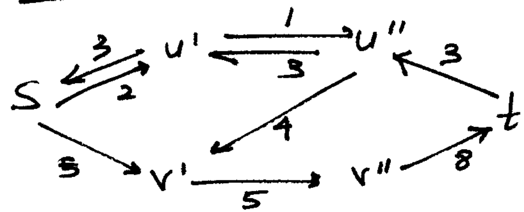


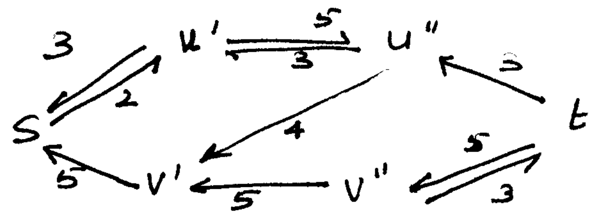
Augmented Paths



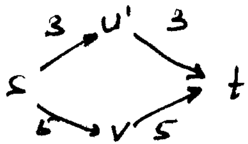


Residual Networks





max flow is $\Rightarrow$



$3 + 5 = 8$.