Q1 a) Say the 2 colors are black and white.

```
for each vertex u G
  if u is not colored
    Color[u] ← black      ← for each start vertex of disjointed subgraph
    Q ← {u}                 set the color to black.
    while {Q is not Empty}
      t ← Dequeue[Q]
      for each v ∈ adj[t]
        if v is not Colored
        Color[v] = ~Color[t]
        Enqueue(Q, v)
```

Complexity = $O(V+E)$

b) Decision Problem.

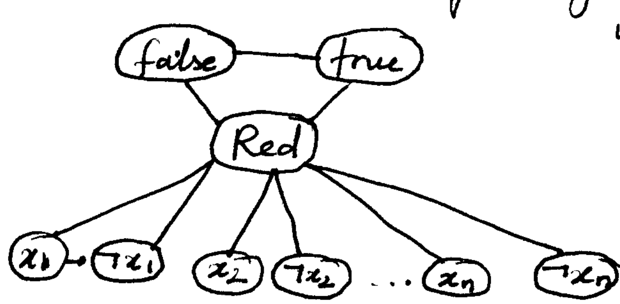
For an undirected graph $G(V,E)$ and a positive integer $k \geq 2$, is there a k-coloring for G?

d) For any truth assignment t for ϕ.

Set $c(x_i) = \phi(x_i)$ $\quad c(\neg x_i) = \phi(\neg x_i)$ $\quad i = 1 \ldots n$

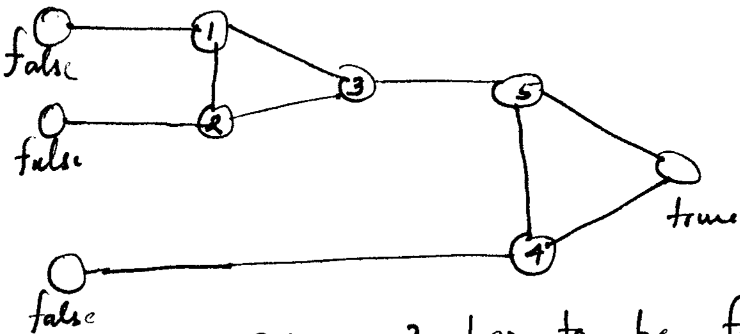
Then one of x_i and $\neg x_i$ is colored c(true) and the other c(false)

Hence the graph G containing only the literal edges is with each ending at 'RED'



This function c is a 3-coloring of G.

e. take the case $x = y = z$ = false and show that it will not give a 3 colorable graph.



Since $x = y$ = false, 3 has to be false. leaving 1,2 with assignment true or red. then we have both z and 3 with false.
But now we have to assign a 'true' vertex with false – Contradiction
∴ Since x, y, z can only be assigned true or false,
then one of them has to be assigned TRUE!

f) 1) 3 Colors is NP. Given a guess we can verify it in polynomial time. Whether $c(u) \neq c(v)$ for every edge $(u,v) \in E$.

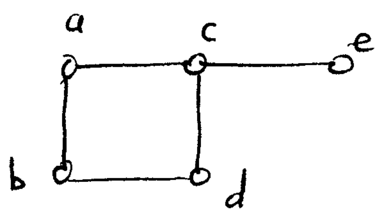
2) To Show 3SAT $\propto$ 3 Color

Transformation as in the textbook. for any clause C_i
Construct a widget with 3 literals in the clause.

Proof of Equivalence:
If there is a truth assignment for ϕ then every $C(i)$ = TRUE.
it means atleast one literal in C_i is true.
Color the 3 special vertices as C(TRUE), C(FALSE) and C(RED)
Color every vertex of variables and negation with
$C(x_i) = C(t(x_i))$ $C(\neg x_i) = C(t(\neg x_i))$
for any widget, if at least one literal is colored C(TRUE)

According to (e) it is 3-Colorable. It means every clause edge satisfies 3-Coloring and color 'clause' edge satisfies 3-Coloring
According to (d) every literal edge satisfies 3-Coloring.
So C is a 3-Coloring of G.
Now we know 3 SAT $\propto$ 3-coloring
and 3 SAT is NPC.
$\therefore$ 3-Coloring is NPC.

Q2 : a)



dominating set $\{a, c\}$

b) Decision Problem:
Does there exist a subset $V' \subseteq V$ such that for $|V'| \leq K$ for every vertex in $(V - V')$ is linked to atleast one vertex in V' such that $e \in E$?

Proof

We choose Vertex Cover (VC) as the source problem and will show that it is a reducible to our dominating set problem. (DSP)

first note that DSP $\in$ NP, since a non deterministic algorithm need to guess only a subset V' of V and check in polynomial time that there exists an edge between $(V - V')$ and V in the original graph.

Now lets transform VC to DSP.

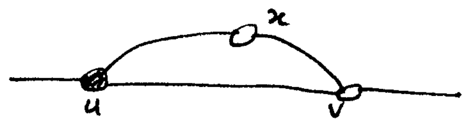
lets say V' is a VC for a graph $G(V, E)$
As per the hint we can add additional vertices to the set V' and it will still remain a VC.

Now $\langle v_1, v_2 \ldots v_k \rangle$ where $|V'|$ = size of VC.

edges $\langle e_1, e_2 \ldots \rangle$ total edges on the graph.

Each edge e_i on VC must be linked with atleast one member in V'.

Now for every edge (u, v) add 2 edges (u, x) and (x, v)



2 Cases to analyse:

Case1: If both u, v are members of VC, then additional vertex x, doesnot affect the VC.

Case2: If only one of (u, v) is in VC, in order to cover edge (u, x) (x, v) we have to chooce atleast one more from u, v, x

Say we choose x

Now after selecting all such vertices, members of $(V - V')$ will have atleast one edge between them and the V' set, since its a VC. But this has however now reduced to a dominating set problem. Reduction complete. $VC \propto DSP$.

But we know that VC is NPC

$\Rightarrow$ $\therefore$ DSP is NPC.