Solutions for HW # 7

① Given a directed graph $G(V,E)$, find a minimum number of vertices, such that eliminating them from G, makes the graph cycle-free.

② for $\forall v \in V$ (i, from 0 to $|V|$)

for each vertex set S with size i ($|S| = i$)

remove S from G and get G'

DFS (G')

if G' has no cycle

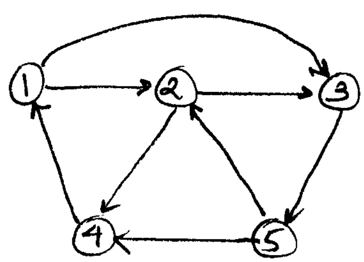
minimum number $\leftarrow i$

FVS $\leftarrow S$

stop.

Complexity : $\sum_{i=0}^{|V|} \binom{|V|}{i} O(|V|+|E|) = 2^{|V|} O(|V|+|E|)$ Exponential.

③



$i = 0, 1$ G' has a cycle.

$i = 2$ $S = \{1, 2\}$ G' has no cycle.

So the minimum number $= 2$

FVS $= \{1, 2\}$

III'ly can veryfy for all other subsets (by inspection) and we conclude that this is the optimum.

④ Heuristic Algorithm,

1. Select a vertex with the highest degree in G.
2. Remove this from G.
3. Apply DFS (G)
4. (If G has no cycle). stop
   else repeat 2 and 3.

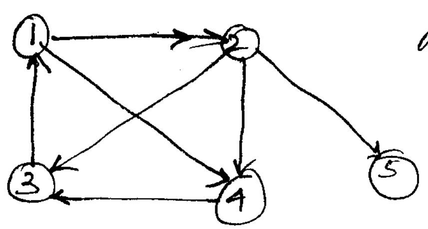
Each iteration has atmost |V| complexity

Selecting vertex : $O(|V|)$ DFS = $O(|V| + |E|)$

total Complexity $= O(|V|) * (O(V) + O(V+E))$

$= O(|V| * (|V| + |E|))$.

⑤



apply heuristic

Select ②

then select ①

FVS = {1, 2} size = 2

But had you selected vertex 1 (size = 1)

It would have been the optimum.

Optimum FVS = {1}

⑥ program & runtime results.