

56:272 Integer Programming & Network Flows
Homework #3 - Due Sept. 24, 1997

1. If you did not complete exercise 2 of HW #2 (Finding resources on the WWW (world-wide-web), include it in this homework.
2. Consider the following zero-one knapsack problem, with a capacity of 12 units of weight:

item #	Weight	Value
1	3	4
2	9	5
3	6	6
4	4	3
5	3	2

Dynamic programming output for this problem is given below:

STAGE 1	<table style="width: 100%; border-collapse: collapse;"> <tr><td style="text-align: right;">s \ x:</td><td style="text-align: center;">0</td><td style="text-align: center;">1</td></tr> <tr><td>0</td><td style="text-align: right;">0.00</td><td style="text-align: right;">-999999.00</td></tr> <tr><td>1</td><td style="text-align: right;">0.00</td><td style="text-align: right;">-999999.00</td></tr> <tr><td>2</td><td style="text-align: right;">0.00</td><td style="text-align: right;">-999999.00</td></tr> <tr><td>3</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>4</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>5</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>6</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>7</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>8</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>9</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>10</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>11</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> <tr><td>12</td><td style="text-align: right;">0.00</td><td style="text-align: right;">4.00</td></tr> </table>	s \ x:	0	1	0	0.00	-999999.00	1	0.00	-999999.00	2	0.00	-999999.00	3	0.00	4.00	4	0.00	4.00	5	0.00	4.00	6	0.00	4.00	7	0.00	4.00	8	0.00	4.00	9	0.00	4.00	10	0.00	4.00	11	0.00	4.00	12	0.00	4.00	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">STATE</th> <th style="text-align: center;">OPTIMAL VALUES</th> <th style="text-align: center;">OPTIMAL DECISIONS</th> <th style="text-align: center;">RESULTING STATE</th> </tr> </thead> <tbody> <tr><td>0</td><td style="text-align: center;">0.00</td><td style="text-align: center;">0</td><td style="text-align: center;">0</td></tr> <tr><td>1</td><td style="text-align: center;">0.00</td><td style="text-align: center;">0</td><td style="text-align: center;">1</td></tr> <tr><td>2</td><td style="text-align: center;">0.00</td><td style="text-align: center;">0</td><td style="text-align: center;">2</td></tr> <tr><td>3</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">0</td></tr> <tr><td>4</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">1</td></tr> <tr><td>5</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">2</td></tr> <tr><td>6</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">3</td></tr> <tr><td>7</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">4</td></tr> <tr><td>8</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">5</td></tr> <tr><td>9</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">6</td></tr> <tr><td>10</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">7</td></tr> <tr><td>11</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">8</td></tr> <tr><td>12</td><td style="text-align: center;">4.00</td><td style="text-align: center;">1</td><td style="text-align: center;">9</td></tr> </tbody> </table>	STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE	0	0.00	0	0	1	0.00	0	1	2	0.00	0	2	3	4.00	1	0	4	4.00	1	1	5	4.00	1	2	6	4.00	1	3	7	4.00	1	4	8	4.00	1	5	9	4.00	1	6	10	4.00	1	7	11	4.00	1	8	12	4.00	1	9
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11	5.00	1	2																																																																																																	
12	9.00	1	3																																																																																																	

STAGE 3

s \ x:	0	1
0	0.00	-999999.00
1	0.00	-999999.00
2	0.00	-999999.00
3	4.00	-999999.00
4	4.00	-999999.00
5	4.00	-999999.00
6	4.00	6.00
7	4.00	6.00
8	4.00	6.00
9	5.00	10.00
10	5.00	10.00
11	5.00	10.00
12	9.00	10.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	6.00	1	0
7	6.00	1	1
8	6.00	1	2
9	10.00	1	3
10	10.00	1	4
11	10.00	1	5
12	10.00	1	6

STAGE 4

s \ x:	0	1
0	0.00	-999999.00
1	0.00	-999999.00
2	0.00	-999999.00
3	4.00	-999999.00
4	4.00	3.00
5	4.00	3.00
6	6.00	3.00
7	6.00	7.00
8		
9	10.00	7.00
10	10.00	9.00
11	10.00	9.00
12	10.00	9.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	6.00	0	6
7	7.00	1	3
8			
9	10.00	0	9
10	10.00	0	10
11	10.00	0	11
12	10.00	0	12

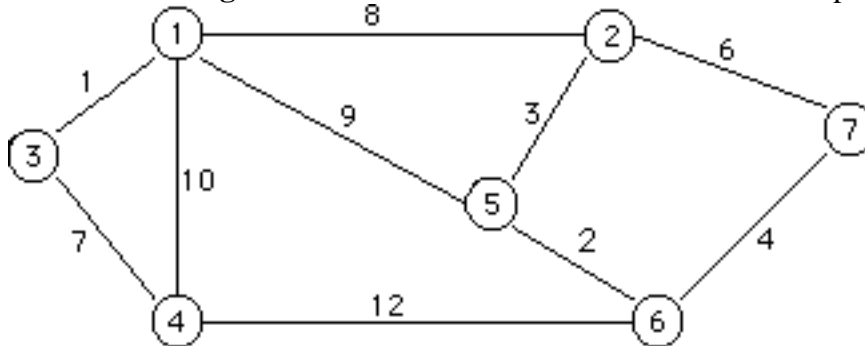
STAGE 5

s \ x:	0	1
0	0.00	-999999.00
1	0.00	-999999.00
2	0.00	-999999.00
3	4.00	2.00
4	4.00	2.00
5	4.00	2.00
6	6.00	6.00
7	7.00	6.00
8	7.00	6.00
9	10.00	8.00
10	10.00	9.00
11	10.00	9.00
12	10.00	12.00

STATE	OPTIMAL VALUES	OPTIMAL DECISIONS	RESULTING STATE
0	0.00	0	0
1	0.00	0	1
2	0.00	0	2
3	4.00	0	3
4	4.00	0	4
5	4.00	0	5
6	6.00	0	6
7	7.00	1	3
8	7.00	0	8
9	10.00	0	9
10	10.00	0	10
11	10.00	0	11
12	12.00	1	9

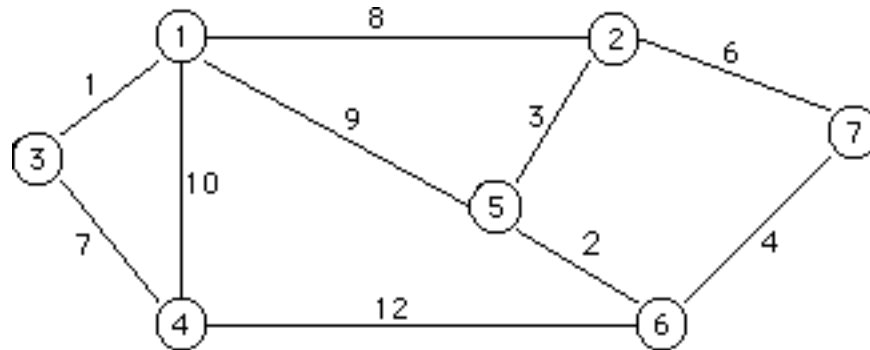
- Do the computations to complete the five blank boxes in stage 4 of the DP output above. Explain the computations:
- What is the optimal solution of this knapsack problem?
- Suppose that the capacity of the knapsack is only 10 pounds. Explain how to use the DP output to determine the optimal knapsack contents. What items should be included in this case?

3. Shortest Path Algorithm. Consider the undirected network for problems #1 & #2:

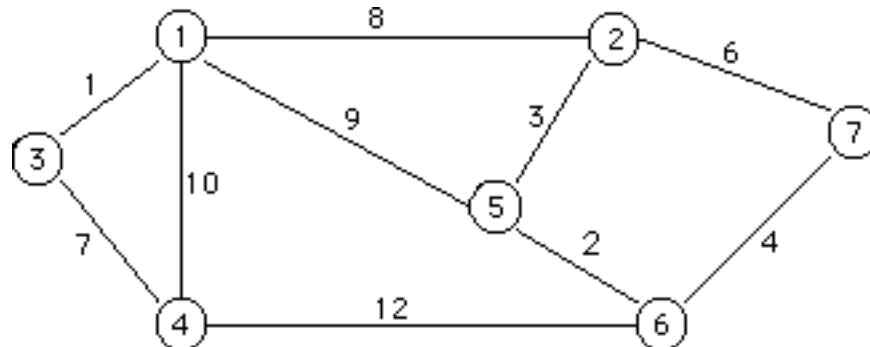


Apply Dijkstra's labeling algorithm to find the shortest path from node 4 to each of the other nodes, where the numbers on the edges are interpreted as lengths. Show the results of each iteration.

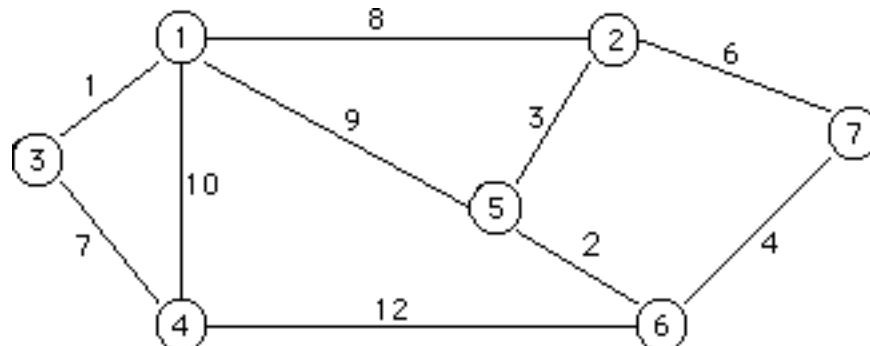
Iteration #1:



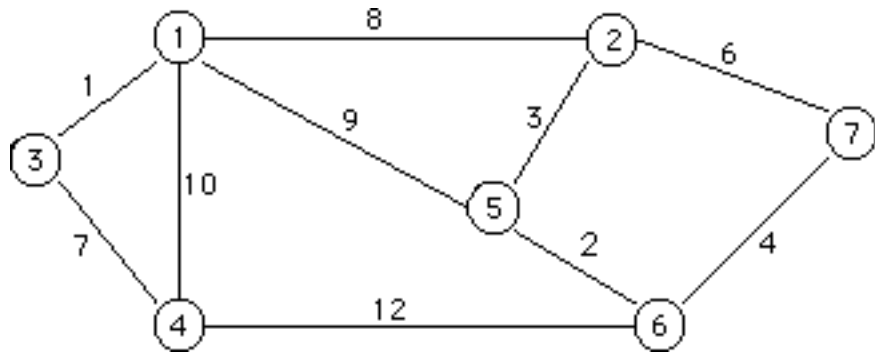
Iteration #2:



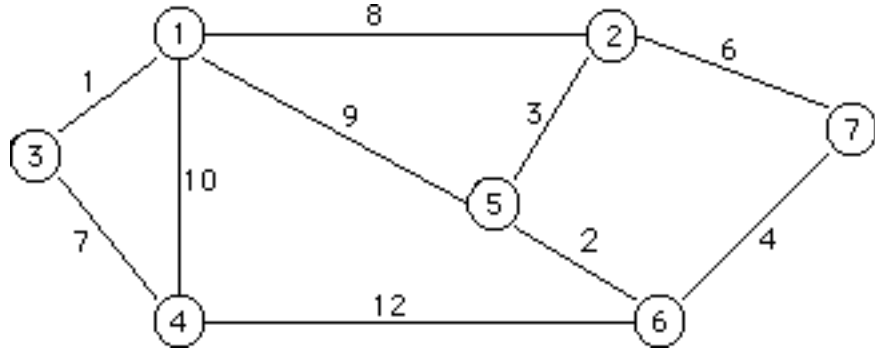
Iteration #3:



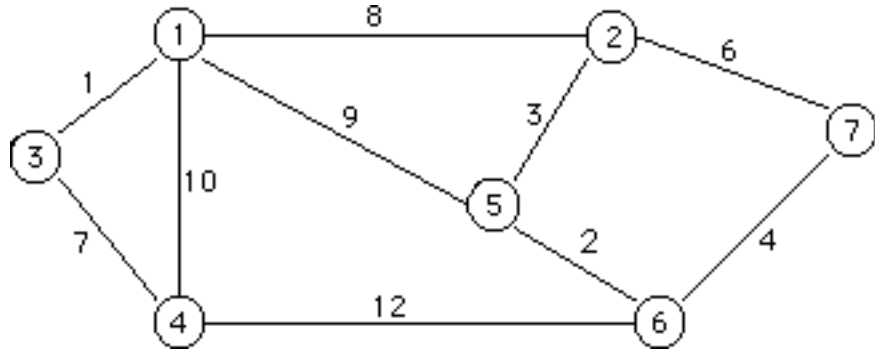
Iteration #4:



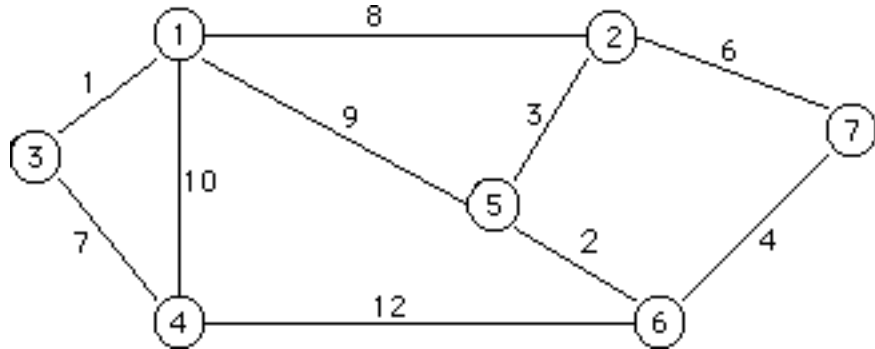
Iteration #5:



Iteration #6:



Iteration #7:



etc.