56:272 Integer Programming & Network Flows
Homework #4 - Due October 8, 1997

1. ***Equipment Replacement:*** The Goode-O'Toole Company runs a machine shop containing an expensive drill press that must be replaced periodically as it wears out. The Vice-President of Manufacturing has just authorized installing a new model, but has asked you to devise an equipment replacement plan for the next seven years, after which a drill press will no longer be needed. Let p_t be the cost of a new model in period t, where

t	1	2	3	4	5	6	7
p_t	100	105	110	115	120	125	130

so that the currently authorized installation incurs the cost 100. Let v_k be the salvage value of a press that is sold at the end of k periods of use, where

k	1	2	3	4	5	6	7
v_k	50	30	15	10	5	2	0

Thus, if the new press is sold at the end of the first period, Goode-O'Toole realizes a revenue of 50; if it is sold at the end of period 2, the revenue is 30; if it is sold at the end of period 7, the revenue is 0.
Let r_k be the operating cost of a piece of equipment during its k^{th} consecutive period of use, where

k	1	2	3	4	5	6	7
r_k	30	40	50	50	60	70	100

Formulate a shortest-path problem to find an optimal replacement policy. (Exercise borrowed, with data revised, from the text *Principles of Operations Research*, by Harvey Wagner.)

2. Inspection Problem: A production line consists of an ordered sequence of **n** production stages each of which consists of a manufacturing operation followed by a potential inspection station. The product being manufactured enters stage 1 in batches of size **B**. As each item of a batch moves along the production line faults may be created in it at any stage **j** with independent probability $\mathbf{b_j}$. A fault created at stage **j** costs $\mathbf{r_{jk}}$ to repair if detected at stage **k**.

Inspection must be carried out at the n^{th} stage, but is optional at other stages. The cost of inspecting at stage **k**, if the batch was last inspected at stage **j**, is $f_{jk} + Bv_{jk}$, where f_{jk} is the fixed cost per batch and v_{jk} is the variable cost per item inspected. How should inspection effort be allocated so as to minimize total costs? *(Example 4.1, page 70, of text by Boffey.)*

Data: n=5, B=2000, b_1=0.025, b_2=0.02, b_3=0.04, b_4=0.04, b_5=0.03.

Inspection costs f_{jk}, v_{jk} tabulated:

j/k	1	2	3	4	5
0	16,29	34,30	39,33	44,38	46,42
1		30,18	34,20	40,23	42,26
2			28,17	33,19	38,22
3				27,13	34,16
4					28,11

(for example, f_{12}=30, v_{12}=18)

Repair costs rjk tabulated:

j/k	1	2	3	4	5
1	0.45	0.90	1.50	2.15	2.60
2		0.50	1.10	1.70	2.10
3			0.70	1.40	1.80
4				0.80	1.35
5					0.6

Model the problem as a shortest path problem, and solve it (either manually or using the **Networks** workspace.)

3. Production Planning. Production by a factory is to be planned for a four-week period in which demand is known with certainty. Production can be done during regular time, overtime, or may be subcontracted to another company. The relevant data are:

Week:	1	2	3	4
Regular time production capacity	100	100	60	100
Regular time cost ($/unit)	15	16	18	20
Overtime production capacity	20	20	10	10
Overtime cost ($/unit)	17	19	21	24
Subcontracting source capacity	40	40	40	40
Subcontracting unit cost	20	21	22	23
Demand Requirements	90	110	100	115

The initial inventory is five units. No shortages are to be planned. The inventory storage cost is $1/unit per week. Formulate a transportation problem (with total supply = total demand) to minimize the sum of shortage and shipping costs. (That is, give the transportation tableau.)

4. Production planning, continued. Suppose next that the demand in exercise 3 above can be backordered, with a penalty of $2.20/unit for each week delay in meeting the demand. (Assume, however, that at the end of week 4, no backorders are allowed.)

- Formulate a transportation problem (with total supply = total demand) to minimize the sum of shortage and shipping costs. (That is, give the transportation tableau.)
- Use the "Northwest-Corner" method to find a feasible solution. What is its cost?
- Use the VAM (Vogel's Approximation Method) to find a feasible solution. What is its cost?
- Find the optimal solution, using the **TRANSPORT** workspace.
- How much lower is the optimal cost, compared to the costs of the solutions found by the NW-corner and VAM methods?