

Energy Output

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Outline

- The wind variation
- Distribution plotting
- The average bottle fallacy
- Mean power of the wind
- Betz' law
- Power density
- Power curves
- The power coefficient
- Calculator guide
- The power calculator
- Annual energy output



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Characterizing Wind Variations

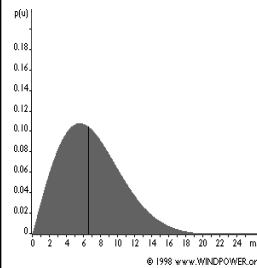
- ✓ Characterization of the variation of wind speed of importance to the wind industry
- ✓ Designers use it to optimize the design of the turbines, e.g., by minimizing the energy generation cost
- ✓ Wind farm designers use it to locate wind farms and select turbines
- ✓ Investors use it to estimate the income from electricity generation



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Wind Variations: Weibull Distribution



The General Pattern of Wind Speed Variations

- ✓ Weibull distribution describes the wind variation for a typical site
- ✓ The site has the mean wind speed of 7 m/s
- ✓ The shape parameter of the function is $k = 2$
- ✓ The data has been collected over a year period



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Weibull Distribution

$$f(x; k, \lambda) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$

where:

$\lambda > 0$ is the scale parameter

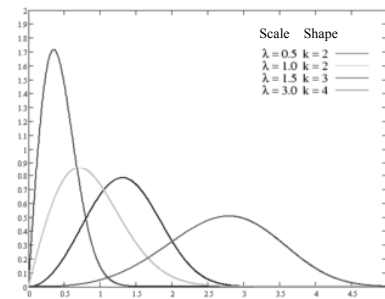
$k > 0$ is the shape parameter of the distribution

For $k = 3.4$, the Weibull distribution appears similar to the normal distribution
 For $k = 1$, the Weibull distribution becomes the exponential distribution



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Weibull Distribution



$$f(x; k, \lambda) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$

Median

$$\lambda \ln(2)^{1/k}$$

Mode

$$\lambda \left(\frac{k-1}{k}\right)^{\frac{1}{k}}$$

Mean

$$\lambda \Gamma\left(1 + \frac{1}{k}\right)$$

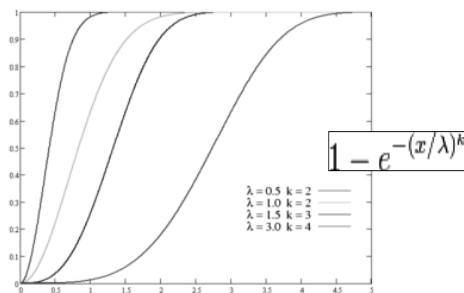
where the gamma function is

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$$



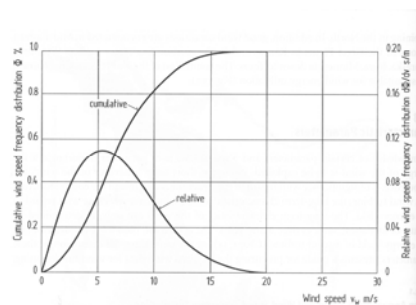
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Cumulative Distribution Function



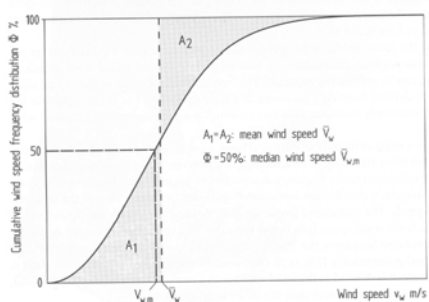
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PDF and CDF



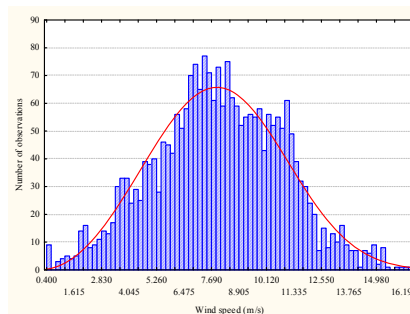
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CDF: Mean vs Median Wind Speed



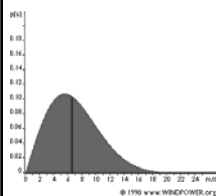
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Wind Speed Histogram



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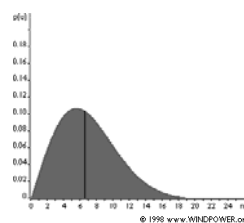
Description of Wind Speeds



- ✓ The area under the pdf curve is always exactly 1 (the probability that the wind is blowing at some wind speed including 0 is 100 %)
- ✓ Median = 6.6 m/s (Half of the blue area is to the left)
- ✓ This means that 50% of the time the wind speed is less than 6.6 m/s, the other 50% of the time it is greater than 6.6 m/s
- ✓ Note: The median 6.6 m/s is not equal the mean 7m/s due to asymmetry of the pdf

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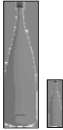
Description of Wind Speeds



- ✓ Mode = 5.5 m/s (The most common wind speed)
- ✓ The statistical distribution of wind speeds depends on location, climate conditions, the landscape, and its surface
- ✓ The Weibull distribution may thus vary in its shape, determined by the pdf parameters

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The Average Bottle Fallacy




- ✓ The average energy content of the wind at a turbine site can not be determined from the average wind speed, rather the Weibull distribution is needed
- ✓ How large (in volume) is the average bottle, one is 0.24m and the other is 0.76m tall and both are of same shape?
- ✓ Though one is only 3.17 taller than the other, its volume is actually $3.17^3 = 32$ times larger than the small bottle (bottle $V = \text{cube of its size}$)

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Betz' Law


The Ideal Braking of the Wind



- ✓ The more kinetic energy a wind turbine extracts from the wind, the more the wind will be slowed down (as it leaves the left side of the turbine in the tunnel)
- ✓ An attempt to extract all the energy from the wind, would reduce the speed to zero, i.e., the air could not leave the turbine
- ✓ In that case we would not extract any energy at all, as the new air would obviously be prevented from entering the rotor of the turbine
- ✓ Passing air without speed change would lead to zero extracted energy

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Betz' Law



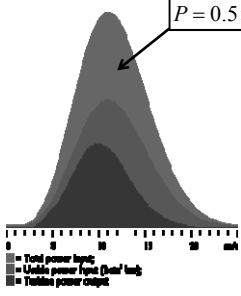
- ✓ The optimal point is: An ideal wind turbine slows down the wind by 2/3 of its original speed ($v_2 = 1/3v_1$)
- ✓ Betz' law (Year 1919) says that one can only convert not more than 16/27 (or 59%) of the kinetic energy in the wind to mechanical energy using a wind turbine. (Albert Betz, German Physicist)

Modern rotors achieve values of the coefficient of performance $C_p = 0.4 - 0.5$, which is 70% to 80% of the theoretically possible value of $C_{pmax} = 0.59$

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Power Density Function

The Power of the Wind

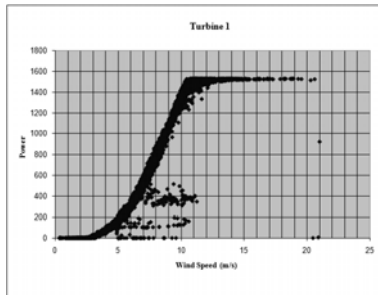
$$P = 0.5 \times \rho \times A \times v^3$$


- ✓ The area under the blue curve shows theoretical power that can be extracted (Betz' law says, 16/27 of the total power in the wind)
- ✓ The total area under the red curve represents the electrical power a certain wind turbine could produce
- ✓ Turbine's power curve determines the actual power produced

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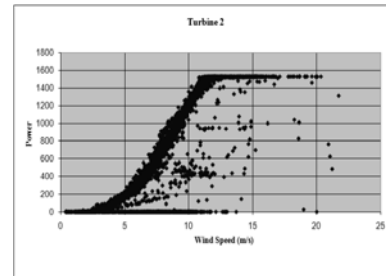
Actual Power Curve



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Actual Power Curve



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Verifying Power Curves

- ✓ Power curves are constructed using measurements in areas with low turbulence intensity, and with the wind coming directly towards the front of the turbine
- ✓ Local turbulence and complex terrain (e.g., turbines placed on a rugged slope) may mean that wind gusts hit the rotor from varying directions
- ✓ It may therefore be difficult to reproduce the power curve exactly at any given location



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Pitfalls of Using Power Curves

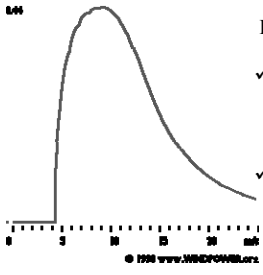
- ✓ The energy content of the wind varies strongly with the wind speed
- ✓ It is important to recognize how that average speed came about, i.e., when the wind varied or it had a relatively constant speed?
- ✓ Most of the wind energy is available at higher wind speeds, e.g., twice the most common wind speed at the site
- ✓ Finally, we need to account for the fact that the turbine may not be running at standard air pressure and temperature, and consequently make corrections for changes in the air density



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The Power Coefficient



Power coefficient = Turbine efficiency

- ✓ The power coefficient indicates how efficiently a turbine converts the wind energy into electricity
- ✓ Power coefficient = $\frac{\text{The electrical power output}}{\text{The wind energy input}}$ (from the power equation)
- ✓ The efficiency varies with the wind speed

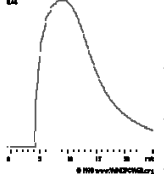
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The Power Coefficient

Power coefficient = Turbine efficiency

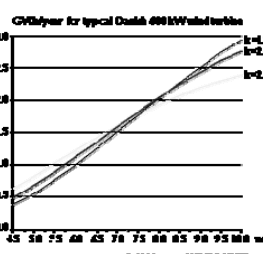
- ✓ Example: A turbine was designed for the max efficiency of 44% at a wind speed of about 9 m/s
- ✓ The efficiency is not as important at low wind speeds as there is not much energy to harvest
- ✓ At high wind speeds the turbine wastes the excess energy above the cut out speed
- ✓ Efficiency therefore matters most in the regions of wind speeds where the most of energy is to be found



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Annual Energy Output from a Wind Turbine



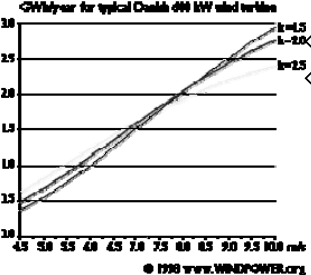
- ✓ The relationship between the average wind speed and the annual energy output from a wind turbine
- ✓ Standard atmosphere with air density of 1.225 kg/m³
- ✓ For each of the Weibull shape parameters $k = 1.5, 2.0,$ and $2.5,$ the annual energy output is computed at different average wind speeds at the turbine hub height

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The Impact of k

GWh/year for typical Double 400 kW wind turbine

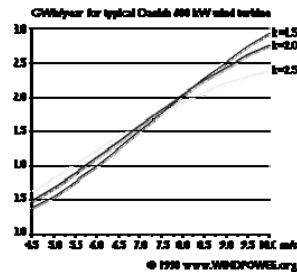


Shape parameter $\gamma = k$
Scale parameter $\lambda = 1$
(Location parameter = 0)

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Annual Energy Output from a Wind Turbine



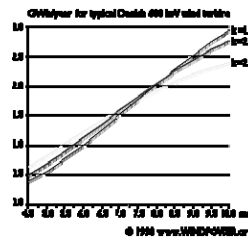
✓ Depending on the value of shape parameter k the energy output may vary up to 50% at a low average wind speed of 4.5 m/s, and some 30% at a higher average wind speed of 10m/s



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Output Varies with the Average Wind Speed



- ✓ Consider the red curve with the shape parameter $k = 2$, normally shown by manufacturers
- ✓ For an average wind speed of 4.5 m/s the machine generates about 0.5 GWh per year
- ✓ For an average wind speed to 9 m/s 2.4 GWh/year it generated
- ✓ Thus, doubling the average wind the energy output increased 4.8 times



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Output Varies with the Wind Speed

- ✓ For the average speeds 5 and 10 m/s (the same speed ratio of 2), energy output would differ by the factor of 4 (rather than 4.8)
- ✓ The reason for the difference, is that the efficiency of the wind turbine varies with the wind speed (the power curve)
- ✓ Note, that the uncertainty that applies to the power curve also applies to the above result
- ✓ The calculations can be refined by considering that, e.g., in temperate climates the wind tends to be stronger in winter than in summer, and stronger during the daytime than at night

Temperate climate: http://www.aee.mmu.ac.uk/ae/Climate/Older/Temperate_Climate.html



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The Capacity Factor

$$\text{Turbine capacity factor} = \frac{\text{The actual annual energy output}}{\text{The theoretical maximum output}}$$

(Theoretical maximum output = if the turbine was running at its maximum rated power during the 8766 hours of the year)

- ✓ Example: If a 600 kW turbine produces 1.5 million kWh in a year, its capacity factor is
 $= 1500000 / (365.25 * 24 * 600) = 1500000 / 5259600$
 $= 0.285 = 28.5\%$
- ✓ The capacity factor may theoretically vary from 0 to 100%, but in practice it usually is 20% to 70%



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Comparison

$$\text{Turbine power coefficient} = \frac{\text{The electrical power output [KW]}}{\text{The wind energy input [KW]}}$$

(Wind farm capacity factor)

$$\text{Turbine capacity factor} = \frac{\text{The actual annual energy output [kWh]}}{\text{The theoretical maximum output [kWh]}}$$

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The Capacity Factor Paradox

- ✓ Although in general a large capacity factor is preferred, it may not always be an economic advantage
- ✓ This is often confusing to those in conventional or nuclear technology
- ✓ In a very windy location, for instance, it may be an advantage to use a larger generator for a given rotor diameter
- ✓ This would tend to lower the capacity factor (using less of the capacity of a relatively larger generator), but it may imply a substantially larger annual production

Non-linear vs quasi-linear capacity relationship:
 Capacity factor (Wind speed) – power produced vs
 Production capacity (Machine speed) – production

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The Capacity Factor Paradox

- ✓ Considering a lower capacity factor for a relatively larger generator, depends both on wind conditions, and on the price of different turbine models
- ✓ Another way of looking at the capacity factor paradox is to say, that to a certain extent a choice can be made between a relatively stable power output (close to the design limit of the generator) with a high capacity factor - or a high energy output (fluctuating) with a low capacity factor

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