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Probabilistic elastic-plastic fracture analysis of circumferentially cracked pipes with finite-length surface flaws

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Abstract

A probabilistic model was developed for predicting elastic-plastic fracture response and reliability of circumferentially cracked pipes with finite-length, constant-depth, internal surface cracks subject to remote bending loads. It involves engineering estimation of energy release rate, J-tearing theory for characterizing ductile fracture, and standard methods of structural reliability theory. The underlying J-estimation model is based on the deformation theory of plasticity, a constitutive law characterized by power law model for stress-strain curve, and an equivalence criterion incorporating reduced thickness analogy for simulating system compliance due to the presence of a crack. New analytical equations were developed to predict J-integral and are shown to be fairly accurate when compared with generally more accurate elastic-plastic finite-element results. Using this J-estimation method, fast probability integrators and simulation methods were formulated to determine the probabilistic characteristics of J. The same methods were used later to predict the probability of crack initiation and net-section collapse as a function of the applied load. Numerical examples are provided to illustrate the proposed methodology. The results show that probabilistic analysis based on net-section collapse (without any margin) may significantly overpredict the reliability of surface-cracked pipes.

1. Introduction

Degraded structural systems comprised of cracked piping systems are often found in nuclear power plants, off-shore drilling platforms, gas transmission lines, fossil power generation plants and others. Of critical importance is the prediction of fracture response and reliability of cracked pipes under various operating conditions, so that appropriate flaw acceptance criteria can be developed. The fracture analysis can be based on linear-elastic or more complex elastic-plastic (nonlinear) models. It is now well established that the nonlinear fracture-mechanics methods provide more realistic measures of fracture behavior of cracked pipes with high toughness and low

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strength materials compared with the elastic methods. In much or all of the working temperature regime of these piping components, the material is being typically stressed above the brittle-to-ductile transition temperature where the fracture response is essentially ductile and the material is capable of considerable inelastic deformation. As such, elastic-plastic theories should be used for fracture analyses of these structural components. While the development is still ongoing, significant progress has been made in the deterministic modeling of both linear-elastic fracture mechanics (LEFM) and elastic-plastic fracture mechanics (EPFM). Probabilistic models have also been developed to estimate various response statistics and reliability (Provan, 1987). Currently, there are many applications of probabilistic fracture mechanics in the field of oil and gas, nuclear, naval, aerospace, and other industries. Nearly all of these methods have been developed strictly based on LEFM models. On the other hand, the probabilistic analysis based on EPFM models has received only a limited attention to date. Probabilistic analyses based on EPFM models are just beginning to appear, particularly, for applications in pressure boundary components (Rahman, 1995; Rahman and Brust, 1995; Rahman et al., 1995; Rahman, 1996, 1997; Rahman and Kim, 1997a; Rahman and Kim 1997b; Rahman, 1998a,b; Rahman and Kim 1998).

In EPFM, the crack-driving force is frequently described in terms of J-integral. The J-integral is an appropriate fracture parameter that describes the crack-tip stress and strain fields adequately when there are no constraint effects. Similar to any deterministic EPFM problem, the evaluation of J-integral for probabilistic analysis can be performed by (1) numerical method and (2) engineering estimation method. Traditionally, a numerical study has been based on elastic-plastic finite element method (FEM). Using FEM, one can calculate J for any crack geometry and load conditions. However, it is also useful to have simplified estimation methods for routine engineering calculations. Accordingly, the probabilistic EPFM analyses based on both methods have been reported (Rahman, 1995; Rahman and Brust, 1995; Rahman et al., 1995; Rahman, 1996, 1997; Rahman and Kim, 1997a; Rahman and Kim 1997b; Rahman, 1998a,b; Rahman and Kim 1998). For example, in the U.S. Nuclear Regulatory Commission's Short Cracks in Piping and Piping Welds Program (Wilkowski et al., 1991), a probabilistic model was developed by the author for elastic-plastic analysis of circumferential through-wall cracks in pipes for leak-before-break applications (Rahman et al., 1995). This model involves a J-estimation method, statistical representation of uncertainties in loads, crack size, and material properties, and first- and second-order reliability methods (FORM/SORM). Shortly thereafter, similar probabilistic models based on other J-estimation formulas have also been reported (Rahman, 1995; Rahman and Brust, 1995; Rahman, 1996, 1997). In these models, the estimation formulas typically consist of closed-form equations of J as a function of load, crack size, and material properties of a structure and hence, do not require any expensive finite element calculations. Actually, this is a major reason why the FORM/SORM algorithms have been successfully developed for probabilistic analysis of elastic-plastic structures. These methods predict reasonably accurate failure probability and have been recently verified by the generally more accurate nonlinear FEM-based probabilistic analysis of through-wall cracks in pipes (Rahman and Kim, 1997a; Rahman and Kim 1997b; Rahman, 1998a,b; Rahman and Kim 1998).

For analyzing circumferentially part-through cracks in pipes by nonlinear FEM, the three-dimensional nature of the surface crack requires immense computational effort to ensure adequate mesh design and enormous computer storage and data reduction. This is quite a formidable task even for a deterministic study that typically requires a large number of analyses to develop robust criteria for pipe flaw evaluations (Krishnaswamy et al., 1995). To alleviate this problem, the use of line-spring/shell model, which essentially reduces the three-dimensional surface-crack problem into a more tractable two-dimensional shell problem, has been suggested by various researchers (Rice, 1972; Rice and Levy, 1972; Shiratori and Miyoshi, 1980; Parks and White, 1982; Lee and Parks, 1995). The line-spring/shell model has been recently claimed to provide fairly accurate predictions of fracture response characteristics with much less computational effort than the three-dimensional finite element analysis (Krishnaswamy et al., 1995; Mohan, 1998). Nevertheless, the need to use FEM for surface cracks raises the degree of complexity in reliability analysis, because the performance function (limit state function) for FORM/SORM is not available as an explicit, closed-form function of input random variables. Although calculating J and other relevant fracture parameters by nonlinear FEM is not unduly difficult, the evaluation of response derivatives or sensitivities (e.g. gradient vector and Hessian matrix of J), required for FORM/SORM analysis, is a formidable task. Using finite-difference method to calculate these derivatives is often computationally expensive, since many repetitions of nonlinear deterministic analysis are required, particularly when there is a large number of random parameters. An attractive alternative, which is used in this study, is to develop approximate analytical methods, which sidestep the need to perform full-scale three-dimensional finite element analysis from the very beginning. If these analytical methods are accurate enough for deterministic calculations, then a corresponding reliability model can be easily developed for probabilistic pipe fracture evaluations.

The objective of this study was to develop a stochastic model for predicting elastic-plastic fracture response and reliability of circumferentially cracked pipes with finite-length, constant-depth, internal surface flaws subject to pure bending loads. It is based on (1) engineering estimation of J-integral, (2) J-tearing theory for characterizing ductile fracture, and (3) standard computational methods of structural reliability theory. The J-estimation method in the underlying deterministic model involves the deformation theory of plasticity, a constitutive law characterized by power law model for stress-strain curve, and an equivalence criterion incorporating reduced thickness analogy for simulating system compliance due to the presence of a crack. The equations for J-integral in a cracked pipe were derived in closed form in terms of elementary functions. Both analytical and simulation methods were formulated to determine the probabilistic characteristics of J. The same methods were used later to predict the probability of crack initiation and net-section collapse as a function of the applied bending load. Numerical examples are presented to illustrate both the deterministic and probabilistic models of the proposed methodology. In the deterministic analysis, the validity of proposed J-estimation formula is evaluated by comparisons with generally more accurate finite-element results. In the probabilistic analysis, the failure probabilities due to initiation of crack growth and net-section collapse are evaluated for a cracked pipe with random material properties and crack size.

2. Elastic-plastic fracture analysis

Consider Fig. 1, which illustrates a pipe with a constant-depth, internal, surface crack symmetrically placed in its bending plane. The pipe has mean radius, R_m, wall thickness, t, and is subject to pure bending moment, M applied at remote ends. The depth and total angle of the surface crack are denoted by a and 2θ, respectively. It is assumed that the well-known Ramberg–Osgood model can represent the material's stress-strain response and is given by

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \tag{1}$$

in which σ_0 is a reference stress, which can be arbitrary, but usually assumed to be the yield stress, E is the modulus of elasticity, $\varepsilon_0 = \sigma_0/E$ is the associated reference strain, and α and n are model parameters usually chosen from best fit of actual laboratory data. Also, the J-resistance curve from the compact-tension specimen is deemed to be adequately characterized by a power-law equation

$$J_R(\Delta a) = J_{Ic} + C\left(\frac{\Delta a}{k}\right)^m \tag{2}$$

in which Δa is the extension of crack depth during crack growth, J_{Ic} is the plane strain mode-I fracture toughness at crack initiation, and C and m are model parameters also obtained from best fit of experimental data. In Eq. (2), k is a dummy

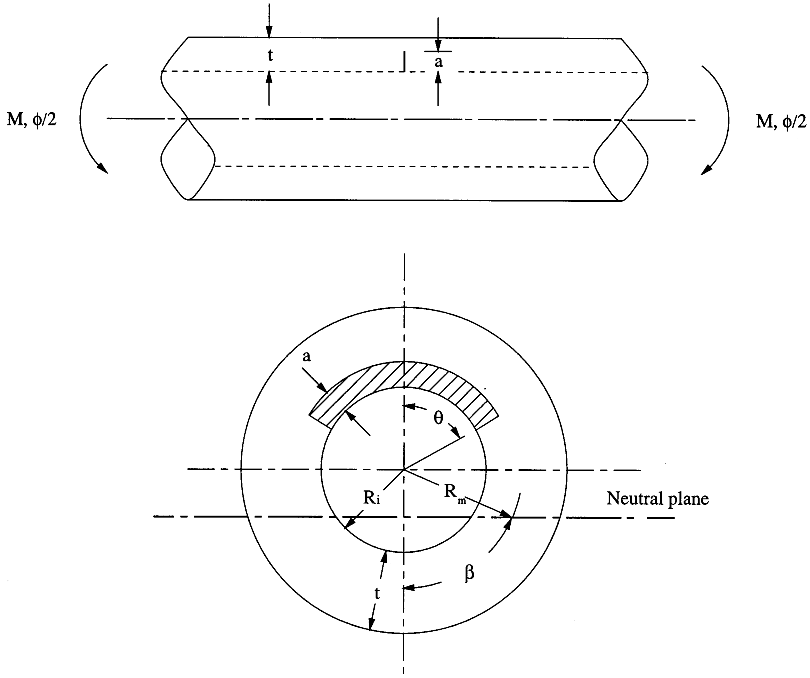

Fig. 1. A pipe with a circumferential surface crack subjected to pure bending.

parameter with a value of 1 introduced here only to dimensionalize C. Note that 'Δa' here is the physical crack extension, i.e. without blunting. This is because blunting is automatically accounted for in the pipe estimation schemes as well as finite element analysis.

2.1. The J-integral

Under elastic-plastic conditions and deformation theory of plasticity when the stress-strain curve is modeled by Eq. (1), the total crack driving force, J, can be obtained by adding the elastic component, J_e and the plastic component, J_p, i.e.

$$J = J_e + J_p \tag{3}$$

For a surface-cracked pipe under pure bending, closed-form expressions can be developed for both J_e and J_p. They are discussed in the following subsections.

2.1.1. Elastic solution

The elastic component, J_e, at the point of maximum depth, is given by (Krishnaswamy et al., 1995; Kumar and German, 1988)

$$J_e = \frac{K_I^2}{E'} \tag{4}$$

where $E' = E/(1-\nu^2)$ for plane strain condition with E and ν representing elastic modulus and Poisson's ratio of the material, respectively, and K_I is the mode-I stress intensity factor. From the LEFM theory, K_I at the deepest point of the crack is given by

$$K_I = \frac{M}{\pi R_m^2 t} F_B(a/t, \theta/\pi)\sqrt{\pi a} \tag{5}$$

in which $F_B(a/t, \theta/\pi)$ is a geometry function relating K_I of a cracked pipe to that for the same size (depth) of a through-wall crack in an infinite plate. In general, F_B should be a function of a/t, θ/π, and R_m/t. But, according to Article IWB-3650 in Section XI of the ASME Code (Evaluation of Flaws in Austenitic Steel Piping, 1986),

$$F_B(a/t, \theta/\pi) = 1.1 + \frac{a}{t}\left[-0.09967 + 5.0057\left(\frac{a}{t}\frac{\theta}{\pi}\right)^{0.565} - 2.8329\left(\frac{a}{t}\frac{\theta}{\pi}\right)\right]. \tag{6}$$

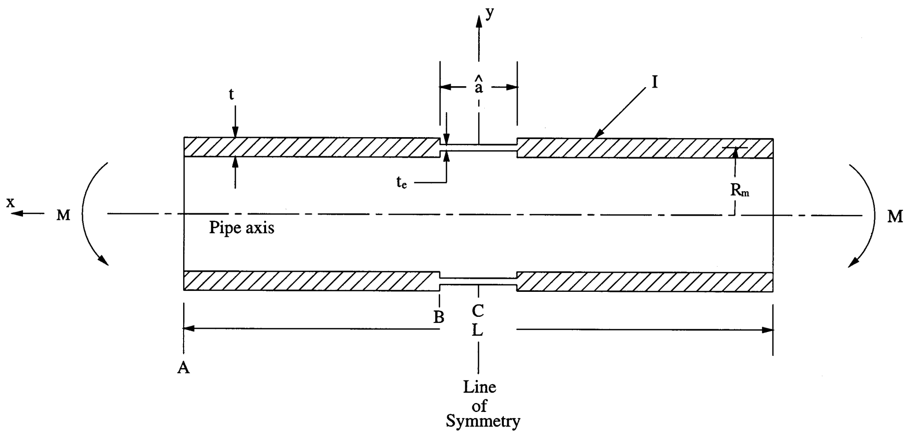

Fig. 2. Reduced section analogy by the SC.ENG1 and SC.ENG2 methods.

Hence, the elastic J is

$$J_e = aF_B(a/t,\,\theta/\pi)^2\,\frac{M^2}{\pi R_m^4 t^2 E'}. \tag{7}$$

2.1.2. Plastic solution

The plastic component, J_p, also at the point of maximum depth, can be defined as

$$J_p = \int_0^M \frac{\partial \phi_p^c}{\partial A}\,dM = \frac{\partial}{\partial A}\int_0^M \phi_p^c\,dM, \tag{8}$$

the evaluation of which requires determination of $M-\phi_p^c$ relationship. In a parallel study, the author used an equivalence approach to determine this relationship by replacing the actual cracked pipe with an uncracked pipe with reduced thickness, t_e, as shown in Fig. 2. Far from the crack plane, the rotation of the pipe is not greatly influenced by whether a crack exists or some other discontinuity is present as long as the discontinuity can approximate the effects of crack. The reduced thickness section, which actually results in material discontinuity, is an attempt to simulate the reduced system compliance due to the presence of crack. Using this approach, the $M-\phi_p^c$ relationship can be estimated to be (Rahman and Brust, 1997)

$$\phi_p^c = \left(\frac{t}{t_e}\right)^{n-1}\left(\frac{\pi}{4\hat{K}}\right)^n \alpha\left(\frac{M}{M_0}\right)^{n-1}\frac{2M}{\pi R_m^4 t^2 E'}\,I_B(a/t,\,\theta/\pi) \tag{9}$$

where

$$\hat{K} = \frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(1+\frac{1}{2n}\right)}{\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}, \tag{10}$$

$$I_B(a/t,\,\theta/\pi) = 2\theta\left(R_m - \frac{t}{2}\right)\int aF_B^2(a/t,\,\theta/\pi)da + 2\theta\int a^2F_B^2(a/t,\,\theta/\pi)da, \tag{11}$$

$$\Gamma(u) = \int_0^\infty \xi^{u-1}\exp(-\xi)d\xi, \tag{12}$$

and $M_0 = \pi R_m^2 t\sigma_0$ is a suitable reference moment. Placing this $M-\phi_p^c$ relationship in Eq. (8) yields

$$J_p = \frac{\alpha(M/M_0)^{n+1}(\pi/4\hat{K})^n}{2t\theta(R_m - t/2 + a)(n+1)}\left[H(a/t)^{n-1}\frac{dG(a/t)}{d(a/t)} + (n-1)G(a/t)H(a/t)^{n-2}\frac{dH(a/t)}{d(a/t)}\right] \tag{13}$$

where

$$H(a/t) = \frac{t}{t_e} \tag{14}$$

and

$$G(a/t) = \frac{2}{\pi R_m^4 t^2 E'}\,I_B(a/t,\,\theta/\pi). \tag{15}$$

Eqs. (7) and (13) provide closed-form solutions of J_e and J_p, respectively. This makes the proposed estimation method computationally feasible and attractive for the development of probabilistic fracture-mechanics models.

2.2. The SC.ENG1 and SC.ENG2 methods

The evaluation of J_p in Eq. (13) requires determination of $H(a/t)$ and $dH(a/t)/d(a/t)$. According to the definition of $H(a/t)$ (see Eq. (14)), this also requires an estimation of the equivalent thickness, t_e for the uncracked pipe. In the equivalence method proposed here t_e can be determined by forcing the net-section-collapse moment of the equivalent uncracked pipe to be equal to the net-section-collapse moment of the actual cracked pipe. For an uncracked pipe with reduced thickness, t_e, the net-section-collapse moment, M_L^d, is

$$M_L^d = 4\sigma_f R_m^2 t_e \tag{16}$$

where σ_f is the flow or collapse stress of the material[1]. However, in determining the net-section-collapse moment, M_L^c, for circumferential surface-cracked pipe, there are several solutions available in the current literature. In this study, two such equations, based on original and Kurihara modifications, were used to determine $H(a/t)$ and its derivative for the evaluation of J_p. Accordingly, the expressions of J_p based on $H(a/t)$ and $dH(a/t)/d(a/t)$ obtained from the original net-section-collapse equations (Kanninen et al., 1976) and Kurihara modification to the net-section-collapse equations (Kurihara et al., 1988) are defined as the SC.ENG1 and the SC.ENG2 methods, respectively. The explicit details for the evaluations of $H(a/t)$ and $dH(a/t)/d(a/t)$ by these two methods are given in the next two subsections.

2.2.1. The SC.ENG1 method

The following are the original equations for the net-section-collapse moment, M_L^c (Kanninen et al., 1976) of a surface-cracked pipe under pure bending and the resulting expressions for $H(a/t)$ and $dH(a/t)/d(a/t)$ used by the SC.ENG1 method. For $\beta \le \pi - \theta$,

$$M_L^c = 2R_m^2 t\sigma_f\left[2\sin\beta - \frac{a}{t}\sin\theta\right] \tag{17}$$

where

$$\beta = \frac{\pi - \theta(a/t)}{2}. \tag{18}$$

When the limit loads from Eqs. (16) and (17) are made equal,

$$H(a/t) = \frac{2}{2\sin\dfrac{\pi-\theta(a/t)}{2} - \dfrac{a}{t}\sin\theta} \tag{19}$$

and

$$\frac{dH(a/t)}{d(a/t)} = \frac{2\left[\theta\cos\dfrac{\pi-\theta(a/t)}{2} + \sin\theta\right]}{\left[2\sin\dfrac{\pi-\theta(a/t)}{2} - \dfrac{a}{t}\sin\theta\right]^2}. \tag{20}$$

For $\beta \ge \pi - \theta$,

$$M_L^c = 2R_m^2 t\sigma_f\left[2 - \frac{a}{t}\right]\sin\beta \tag{21}$$

where

$$\beta = \frac{\pi(1-a/t)}{2-a/t}. \tag{22}$$

When the limit loads from Eqs. (16) and (21) are made equal,

$$H(a/t) = \frac{2}{(2-a/t)\sin\dfrac{\pi(1-a/t)}{2-a/t}} \tag{23}$$

and

$$\frac{dH(a/t)}{d(a/t)} = \frac{\dfrac{2\pi}{2-a/t}\cos\dfrac{\pi(1-a/t)}{2-a/t} + 2\sin\dfrac{\pi(1-a/t)}{2-a/t}}{\left[(2-a/t)\sin\dfrac{\pi(1-a/t)}{2-a/t}\right]^2}. \tag{24}$$

[1] The flow stress is a stress value between the material's yield strength and ultimate strength. Typically, it is assumed to be the average of yield and ultimate strengths of a material.

2.2.2. The SC.ENG2 method

The following are the Kurihara modifications (Kurihara et al., 1988) to the equations for the net-section-collapse moment, M_L^c of a surface-cracked pipe under pure bending and the resulting expressions for $H(a/t)$ and $dH(a/t)/d(a/t)$ used by the SC.ENG2 method. For $\beta \leq \pi - \theta$,

$$M_L^c = 2R_m^2 t\sigma_f\left[2m\sin\beta + \left(1 - \frac{a}{t} - m\right)\sin\theta\right] \tag{25}$$

where

$$\beta = \frac{\pi}{2} + \frac{\theta(1 - a/t - m)}{2m}. \tag{26}$$

When the limit loads from Eqs. (16) and (25) are made equal,

$$H(a/t) = \frac{2}{K_1(a/t)} \tag{27}$$

and

$$\frac{dH(a/t)}{d(a/t)} = -\frac{2}{K_1(a/t)^2}\frac{dK_1(a/t)}{d(a/t)} \tag{28}$$

where

$$K_1(a/t) = 2m\sin\left[\frac{\pi}{2} + \frac{\theta(1 - a/t - m)}{2m}\right] + (1 - a/t - m)\sin\theta \tag{29}$$

and

$$\frac{dK_1(a/t)}{d(a/t)} = -\frac{1}{m}\cos\left[\frac{\pi}{2} + \frac{\theta(1 - a/t - m)}{2m}\right]\left[m\theta + \theta(1 - a/t)\frac{\partial m}{\partial(a/t)}\right] + \frac{2\partial m}{\partial(a/t)}\sin\left[\frac{\pi}{2} + \frac{\theta(1 - a/t - m)}{2m}\right] - \left[1 + \frac{\partial m}{\partial(a/t)}\right]\sin\theta. \tag{30}$$

For $\beta \geq \pi - \theta$

$$M_L^c = 2R_m^2 t\sigma_f\left[1 - \frac{a}{t} + m\right]\sin\beta \tag{31}$$

where

$$\beta = \frac{\pi(1 - a/t)}{1 - a/t + m}. \tag{32}$$

When the limit loads from Eqs. (16) and (31) are made equal,

$$H(a/t) = \frac{2}{K_2(a/t)} \tag{33}$$

and

$$\frac{dH(a/t)}{d(a/t)} = -\frac{2}{K_2(a/t)^2}\frac{dK_2(a/t)}{d(a/t)} \tag{34}$$

where

$$K_2(a/t) = Q_1(a/t)\sin Q_2(a/t) \tag{35}$$

$$\frac{dK_2(a/t)}{d(a/t)} = Q_1\cos Q_2\frac{dQ_2}{d(a/t)} + \sin Q_2\frac{dQ_1}{d(a/t)} \tag{36}$$

$$Q_1(a/t) = 1 - \frac{a}{t} + m \tag{37}$$

$$Q_2(a/t) = \frac{\pi(1 - a/t)}{Q_1(a/t)} \tag{38}$$

$$\frac{dQ_1}{d(a/t)} = -1 + \frac{\partial m}{\partial(a/t)} \tag{39}$$

and

$$\frac{dQ_2}{d(a/t)} = -\frac{\pi Q_1 + \pi(1 - a/t)\frac{dQ_1}{d(a/t)}}{Q_1^2}. \tag{40}$$

In Eq. (25) through to Eq. (40), the functions $m(a/t, \theta/\pi)$ and $\partial m(a/t, \theta/\pi)/\partial(a/t)$ are defined as

$$m(a/t, \theta/\pi) = 1 - (a/t)^2(\theta/\pi)^{0.2} \tag{41}$$

$$\frac{\partial m(a/t, \theta/\pi)}{\partial(a/t)} = -2(a/t)(\theta/\pi)^{0.2} \tag{42}$$

which are empirical functions developed by Kurihara et al. (1988). When the exponents in the equation for $m(a/t, \theta/\pi)$ are chosen to be 2 and 0.2 (see Eq. (41)), the predicted net-section-collapse moments of pipes with both short and long deep flaws compare reasonably well with the experimental data (Kurihara et al., 1988). When these exponents are assigned large positive values, m approaches 1 and the resulting Kurihara modifications to the net-section-collapse equations degenerate to the original equations. In that case, the difference between the SC.ENG1 and

SC.ENG2 methods also vanishes. In this study, however, the exponents suggested by Kurihara et al. (i.e. 2 and 0.2) were used for the development of SC.ENG2 method and subsequent numerical calculations presented in this paper.

2.3. Deterministic evaluation of SC.ENG1 and SC.ENG2 methods

To evaluate the accuracy of the SC.ENG1 and SC.ENG2 methods, the J-integral solutions for several surface-cracked pipes under pure bending loads were compared with finite-element results from author's previous work (Rahman and Brust, 1997). Four pipe cases were considered. In all cases, the outer pipe diameter, $D_o = 404.2$ mm (15.91 inches) and pipe wall thickness, $t = 26.42$ mm (1.04 inches). The stress-strain curve was idealized with the Ramberg–Osgood model (i.e. Eq. (1)). Two different crack lengths, such as $\theta/\pi = 1/4$ (long crack) and $\theta/\pi = 1/16$ (short crack), and two different hardening exponents, such as $n = 3$ and $n = 10$, were selected. In all cases, the crack depth ratio, a/t was fixed to be 50 percent. Also, the following values were assumed: $\sigma_0 = 241$ MPa (35 ksi), $\alpha = 1$, $E = 207$ GPa (30,000 ksi), and $\nu = 0.29$.

Figs. 3–6 show the plots of J versus M obtained by the proposed SC.ENG1 and SC.ENG2 methods and FEM. The FEM solutions were obtained by using the ABAQUS computer code (Version 5.3) with the line-spring/shell elements. Finite-element results from the line-spring/shell model were previously validated against those from the full-scale three-dimensional model (Krishnaswamy et al., 1995). The comparisons with the FEM results suggest that the SC.ENG2 method provides reasonably accurate estimates of J-integral for various applied loads for various combinations of crack length (θ/π) and material constant (n). The agreement between the SC.ENG2 and FEM solutions is excellent when θ/π or n is larger. Figs. 3–6 also show the corresponding results from the SC.ENG1 method which predicted smaller J compared with both SC.ENG2 and finite-element solutions for any given moment. Since, the Kurihara modification in SC.ENG2 method lowers the Net-Section-Collapse load from the original equations, the equivalent thickness, t_e is larger in SC.ENG1 method than that in SC.ENG2 method. Therefore, in the SC.ENG1 method, the values of $H(a/t)$ [recall, $H(a/t) = t/t_e$] would be smaller resulting in smaller J_p by the SC.ENG1 method as compared with that by the SC.ENG2 method (see Eq. (13)).

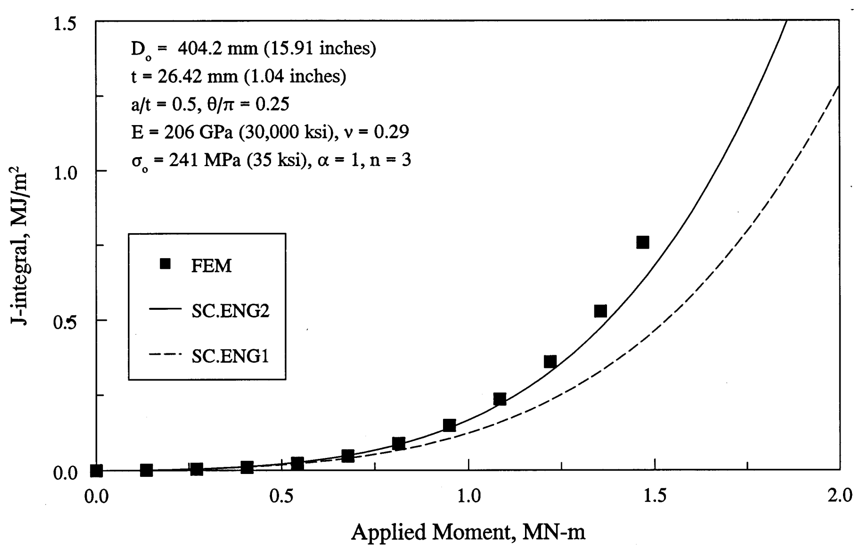

Fig. 3. Predicted J by the proposed methods and FEM ($\theta/\pi = 1/4$, $n = 3$).

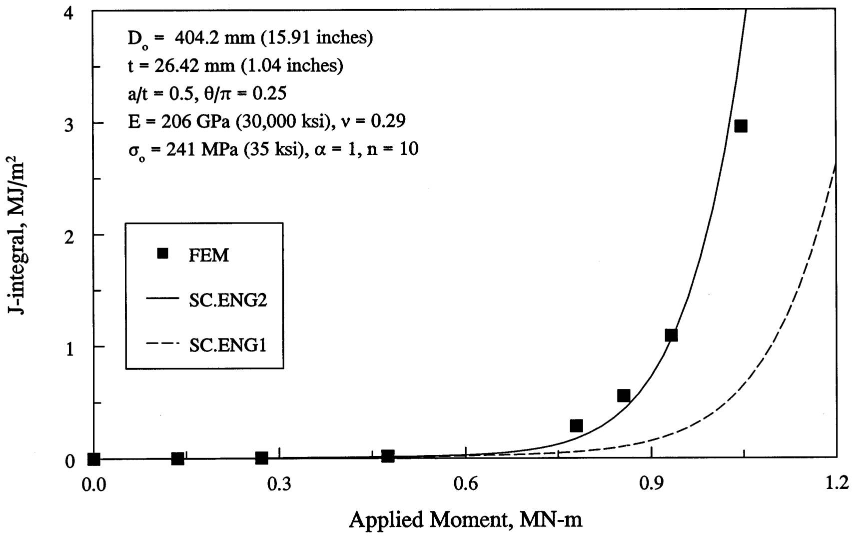

Fig. 4. Predicted J by the proposed methods and FEM ($\theta/\pi = 1/4$, $n = 10$).

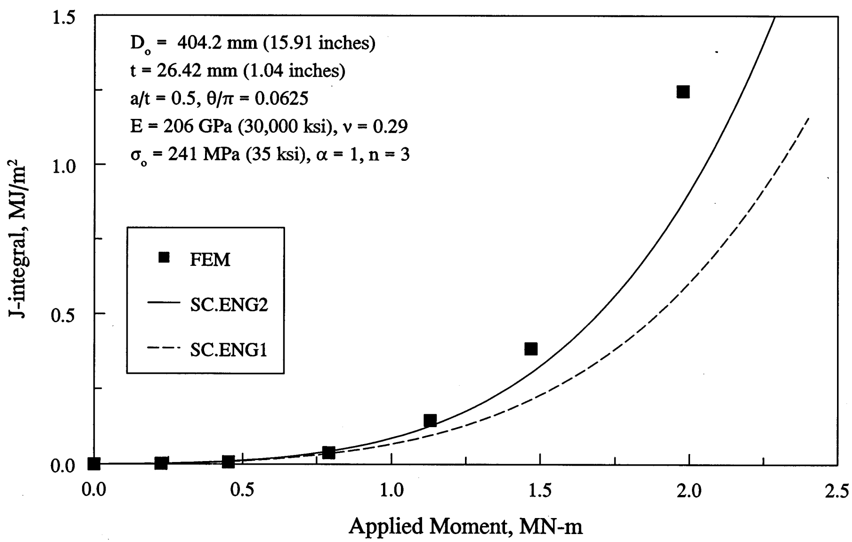

Fig. 5. Predicted J by the proposed methods and FEM ($\theta/\pi = 1/16$, $n = 3$).

Hence, the trend shown in these figures is expected. Since, the SC.ENG2 method provided better J predictions than the SC.ENG1 method, it was decided to use the SC.ENG2 method for conducting subsequent probabilistic analyses to be discussed in the forthcoming sections.

Note that the evaluations of J-integral predicted by the proposed SC.ENG1 and SC.ENG2 methods, which are presented in this paper, are limited to the specific pipe geometry, crack size, and material constants defined above. More numerical results from FEM are needed to systematically verify the accuracy of the proposed methods. They are subjects of current research by the author.

3. Failure load

In order to evaluate structural integrity, it is required to know the load-carrying capacity of a piping system. There are several means by which it can be estimated. They are based on various definitions of failure criteria such as, initiation of crack growth and unstable crack growth in elastic-plastic fracture mechanics, and the Net-Section-Collapse in limit-load analysis. They are briefly described in the following subsections.

3.1. Initiation load

The initiation load, M_i can be defined as the bending moment which corresponds to initiation of crack growth in a pipe. If J is a relevant crack-driving force, it can be estimated by solving the following nonlinear equation

$$J(M_i, a_0) - J_{Ic} = 0 \tag{43}$$

in which $J(M_i, a_0)$ is the energy release rate (ie. J-integral) for load M_i and initial crack depth a_0, which can be obtained from Eqs. (3), (7) and (13), and J_{Ic} is the plane strain mode-I fracture toughness at crack initiation. Standard numerical methods, such as the bisection method, Newton–Raphson method, and others, can be applied to solve Eq. (43) (Press et al., 1990).

3.2. Maximum or instability load

In applications of nonlinear fracture mechanics, the J-tearing theory is a very prominent concept for calculating maximum load-carrying capacity of a pipe. It is based on the fact that fracture instability can occur after some amount of stable crack growth in tough and ductile materials with an attendant higher applied load level at fracture. Let J and J_R $(a - a_0)$ denote the crack-driving force and fracture toughness of a ductile piping material as a function of load and crack-depth extension, respectively. The limit state characterizing fracture instability based on J-tearing theory is given by

$$J(M_{\max}, a^*) - J_R(a^* - a_0) = 0 \tag{44}$$

$$\frac{\partial J}{\partial a}(M_{\max}, a^*) - \frac{\partial J_R}{\partial a}(a^* - a) = 0 \tag{45}$$

where $M_{\max}$ and a^* are load and crack depth when crack growth becomes unstable. Eqs. (44) and (45) are two nonlinear simultaneous equations with the independent variables $M_{\max}$ and a^*. Again, they can be solved by standard methods, such as the Newton–Raphson method (Press et al., 1990).

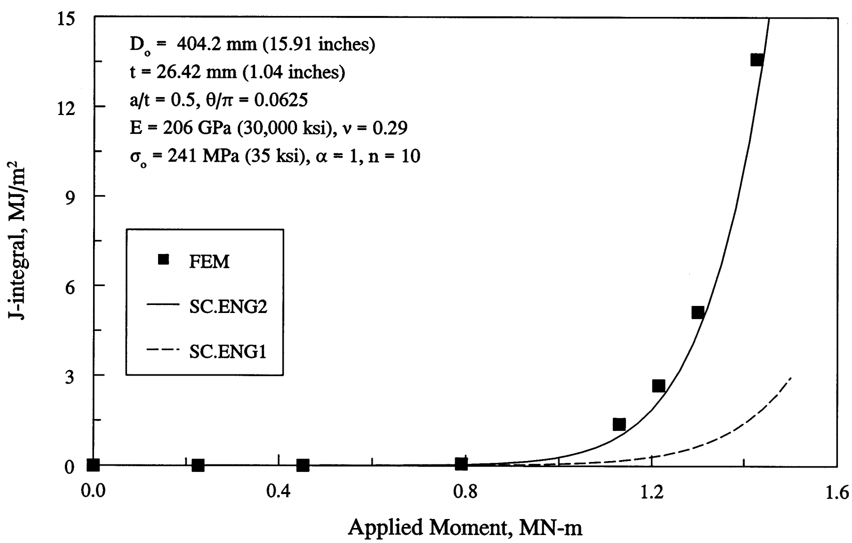

Fig. 6. Predicted J by the proposed methods and FEM ($\theta/\pi = 1/16$, $n = 10$).

Note, the J-resistance curve, which is typically generated from small-scale laboratory specimens, must be prescribed in such a way that similar constraint conditions exist in both cracked structure and laboratory specimens. Otherwise, constraint effects should be accounted for in the fracture-mechanics analysis.

3.3. Net-section-collapse load

The Net-Section-Collapse analysis is a simple and straightforward method for predicting failure load of a cracked pipe. In this analysis, it is assumed that (1) the failure load occurs when the pipe section containing the crack becomes fully plastic, (2) there is insignificant crack growth from crack initiation to failure, and (3) the toughness of the material is sufficiently high so that failure is governed by the strength of materials (i.e. the flow stress). The flow stress is a value between the yield and ultimate strengths of a material and represents an average critical net-section stress throughout the flawed ligament of the structure. Based on this assumptions, the original Net-Section-Collapse load, M_{nsc}, is given by (Rahman, 1998; Rahman and Wilkowski, 1998)

$$M_{\text{nsc}} = \begin{matrix} 2\sigma_f R_m^2 t\left[2\sin\beta - \dfrac{a}{t}\sin\theta\right], & \beta \le \pi - \theta \\ 2\sigma_f R_m^2\left[2 - \dfrac{a}{t}\right]\sin\beta, & \beta \ge \pi - \theta \end{matrix} \tag{46}$$

where,

$$\beta = \begin{cases} \dfrac{\pi - \theta\dfrac{a}{t}}{2}, & \beta \le \pi - \theta \\ \dfrac{\pi\left(1 - \dfrac{a}{t}\right)}{2 - \dfrac{a}{t}}, & \beta \ge \pi - \theta \end{cases} \tag{47}$$

which is a repetition of Eqs. (17) and (21). Previously, the flow stress σ_f was not needed to be quantified to calculate $H(a/t)$ for the SC.ENG1 and SC.ENG2 methods. For calculating Net-Section-Collapse moment of a pipe, however, it must be specified explicitly. In this paper, σ_f was assumed to be the average of yield and ultimate strengths of the pipe material. Note that the alternative limit-load equations based on Kurihara modifications (see Eqs. (25) and (31)) could have been used, but they were not used here for calculating the Net-Section-Collapse moment explicitly. This is consistent with the load calculations in the ASME Section XI Code (Rahman and Brust, 1997).

4. Probabilistic fracture mechanics and reliability

4.1. Random parameters and fracture response

Consider a cracked pipe with uncertain mechanical and geometric characteristics that is subject to random loads. Denote by $\boldsymbol{X}$ an N-dimensional random vector with components $X_1, X_2, \ldots, X_N$ characterizing all uncertainty in the system and load parameters. For example, when a surface-cracked pipe is considered, the possible random components are: crack length ratio, θ/π, initial crack depth ratio, a_0/t, pipe radius-to-thickness ratio, R_m/t, elastic modulus, E, basic tensile strength parameters, σ_y and σ_u, Ramberg-Osgood constitutive parameters, α and n, fracture toughness parameters, J_{Ic}, C, and m, and applied bending moment, M. All or some of these variables can be modeled as random variables. Hence, any relevant response, such as the J-integral, should be evaluated by the probability

$$F_J(j_0) \overset{\text{def}}{=} \Pr[J(\boldsymbol{X}) < j_0] \overset{\text{def}}{=} \int_{J(x) < j_0} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} \tag{48}$$

or the probability density $f_J(j_0) = dF_J(j_0)/dj_0$, where $F_J(j_0)$ is the cumulative probability distribution function of J and $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the known joint probability density function of $\boldsymbol{X}$.

The above fracture parameter J can also be applied to determine load-carrying capacity of surface-cracked pipes. Several fracture criteria based on this J-integral parameter and Net-Section-Collapse are discussed in this paper. In a generic sense, let $M_f(\boldsymbol{X})$ denote the failure moment for a given surface-cracked pipe under pure bending. Note that $M_f(\boldsymbol{X})$ is always random because it depends on input vector $\boldsymbol{X}$ which is random. It can be evaluated when a relevant crack driving force from deterministic fracture (e.g. J-integral from finite element analysis or Eqs. (7) and (13)) and an appropriate fracture criteria (e.g. Eq. (43) or Eqs. (44) and (45)) are known. Suppose that the design requires $M_f(\boldsymbol{X})$ to be always greater than the applied load M (M can be random as well). This requirement cannot be satisfied with certainty because both the system and the load parameters are uncertain. Hence, the performance of the cracked pipe should be evaluated by the reliability P_S or its complement, the probability of failure, $P_F(P_S = 1 - P_F)$ defined as

$$P_F \overset{\text{def}}{=} \Pr[g(\boldsymbol{X}) < 0] \overset{\text{def}}{=} \int_{g(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \tag{49}$$

where $g(\boldsymbol{X})$ is the performance function given by

$$\begin{aligned} g(\boldsymbol{X}) &= M_f - M \\ &= h(\sigma_y, \sigma_u, \alpha, n, J_{Ic}, C, m, \theta/\pi, a_0/t, R_m/t, E) \\ &\quad - M \end{aligned} \tag{50}$$

in which h is a function (implicit) of random parameters characterizing pipe's structural resistance (only the random arguments are shown in Eq. (50)). The failure probability, defined by Eq. (49), can be evaluated if the appropriate fracture criterion is known. For example, when $M_f(\boldsymbol{X})$ is equal to the initiation load $M_i(\boldsymbol{X})$, P_F in Eq. (49) corresponds to the probability of initiation of crack growth which may provide a conservative estimate of pipe's structural performance. A more realistic evaluation of pipe's reliability can be evaluated if $M_f(\boldsymbol{X})$ is equal to maximum load $M_{max}(\boldsymbol{X})$ (which allows the crack to grow until it becomes unstable or through-wall) in which case P_F represents failure probability due to the exceedance of pipe's maximum load-carrying capacity. When EPFM-based failure criteria are not necessary, simple performance function based on limit-load analysis [i.e., $M_f(\boldsymbol{X}) = M_{nsc}(\boldsymbol{X})$] can also be used to determine failure probability of pipes.

Numerical efforts are often required to compute $M_f(\boldsymbol{X})$, particularly when $M_f(\boldsymbol{X}) = M_i(\boldsymbol{X})$ or $M_{max}(\boldsymbol{X})$. This is true in spite of analytical representation of J_e- and J_p-integrals. In this paper, the Newton-Raphson method was used to conduct the numerical analysis.

4.2. Structural reliability analysis

The generic expression for both probabilities in Eqs. (48) and (49) involves a multidimensional probability integration for their evaluation. In this study, standard reliability methods, such as first- and second-order reliability methods (Hasofer and Lind, 1974; Rackwitz and Fiessler, 1978; Ditlevsen, 1979a,b; Fiessler et al., 1979; Chen and Lind, 1982; Breitung, 1984; Der Kiureghian and Liu, 1986; Madsen et al., 1986; Der Kiureghian et al., 1987; Hohenbichler et al., 1987; Breitung, 1989; Tvedt Tvedt, 1990; Haldar and Mahadevan, 1995), Monte Carlo with importance sampling (MCIS) (Harbitz, 1986; Hohenbichler, 1988; Ibrahim and Rahman, 1991), and direct Monte Carlo simulation (MCS) (Rubinstein, 1981), were used to compute these probabilities. They are briefly described here to compute the probability of failure P_F in Eq. (49) assuming a generic N-dimensional random vector $\boldsymbol{X}$ and the performance function $g(\boldsymbol{x})$ defined by Eq. (50). The same methods can be applied to determine the probability $F_J(j_0)$ defined by Eq. (48).

4.2.1. First- and second-order reliability methods

The first- and second-order reliability methods are based on linear (first-order) and quadratic (second-order) approximations of the limit state surface $g(\boldsymbol{x}) = 0$ tangent to the closest point of the surface to the origin of the space. The determination of this point involves nonlinear constrained optimization and is usually performed in the standard Gaussian image of the original space. The FORM/SORM algorithms involve several steps. First, the space $\boldsymbol{x}$ of uncertain parameters $\boldsymbol{X}$ is transformed into a new N-dimensional space $\boldsymbol{u}$ consisting of independent standard Gaussian variables $\boldsymbol{U}$. The original limit state $g(\boldsymbol{x}) = 0$ then becomes mapped into the new limit state $g_U(\boldsymbol{u}) = 0$ in the u space. Second, the point on the limit state $g_U(\boldsymbol{u}) = 0$ having the shortest distance to the origin of the u space is determined by using an appropriate nonlinear optimization algorithm. This point is referred to as the design point or beta point, and has a distance β_{HL} (known as reliability index) to the origin of the u space. Third, the limit state $g_U(\boldsymbol{u})=0$ is approximated by a surface tangent to it at the design point. Let such limit states be $g_L(\boldsymbol{u})=0$ and $g_Q(\boldsymbol{u})=0$, which correspond to approximating surfaces as hyperplane (linear or first-order) and hyperparaboloid (quadratic or second-order), respectively. The probability of failure P_F (Eq. (49)) is thus approximated by $\Pr[g_L(\boldsymbol{U})<0]$ in FORM and $\Pr\lfloor g_Q(\boldsymbol{U})<0\rfloor$ in FORM. These first-order and second-order estimates $P_{F,1}$ and $P_{F,2}$ are given by (Hasofer and Lind, 1974; Rackwitz and Fiessler, 1978; Ditlevsen, 1979a,b; Fiessler et al., 1979; Chen and Lind, 1982; Breitung, 1984; Der Kiureghian and Liu, 1986; Madsen et al., 1986; Der Kiureghian et al., 1987; Hohenbichler et al., 1987; Breitung, 1989; Tvedt; Tvedt, 1990; Haldar and Mahadevan, 1995)

$$P_{F,1}=\Phi(-\beta_{HL}) \tag{51}$$

$$P_{F,2}\cong\Phi(-\beta_{HL})\prod_{i=1}^{N-1}\left(1-\kappa_i\frac{\phi(-\beta_{HL})}{\Phi(-\beta_{HL})}\right)^{-1/2} \tag{52}$$

where

$$\phi(u)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}u^2\right) \tag{53}$$

and

$$\Phi(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u}\exp\left(-\frac{1}{2}\xi^2\right)d\xi \tag{54}$$

are the probability density and cumulative distribution functions, respectively, of a standard Gaussian random variable, and κ_i is the ith principal curvatures of the limit state surface at the design point. Further details of FORM/SORM equations are available in the following references, Hasofer and Lind (1974), Rackwitz and Fiessler (1978), Ditlevsen (1979a,b), Fiessler et al. (1979), Chen and Lind (1982), Breitung (1984), Der Kiureghian and Liu (1986), Madsen et al. (1986), Der Kiureghian et al. (1987), Hohenbichler et al. (1987), Breitung (1989), Tvedt; Tvedt (1990), Haldar and Mahadevan (1995).

FORM/SORM are standard computational methods of structural reliability theory. In this study, a modified HL-RF algorithm, described in Appendix A, was used to solve the associated optimization problem. The first- and second-order sensitivities were calculated numerically by the finite difference method.

4.2.2. Monte Carlo simulation

Consider a generic N-dimensional random vector $\boldsymbol{X}$ which characterizes uncertainty in all load and system parameters with the known joint distribution function, $F_{\boldsymbol{X}}(\boldsymbol{x})$. Suppose that $\boldsymbol{x}^{(1)}$, $\boldsymbol{x}^{(2)}$,..., $\boldsymbol{x}^{(L)}$ are L realizations of input random vector, $\boldsymbol{X}$, which can be generated independently. Rubinstein (1981) provides a simple method to generate $\boldsymbol{X}$ from its known probability distribution. Let $g^{(1)}$, $g^{(2)}$,..., $g^{(L)}$ be the output samples of $g(\boldsymbol{X})$ corresponding to the input $\boldsymbol{x}^{(1)}$, $\boldsymbol{x}^{(2)}$,..., $\boldsymbol{x}^{(L)}$ that can be obtained by conducting repeated deterministic evaluation of the performance function in Eq. (50). Define L_f as the number of trials, which are associated with negative values of the performance function. Then, the estimate $P_{F,MCS}$ by simulation is

$$P_{F,MCS}=\frac{L_f}{L}, \tag{55}$$

which approaches the exact failure probability P_F when L approaches infinity. When L is finite, a statistical estimate on the probability estimator may be needed. In general, the required sample size must be at least $10/\mathrm{Min}(P_F,\ 1-P_F)$ for a 30% coefficient of variation of the estimator

4.2.3. Monte Carlo with importance sampling

In Monte Carlo with importance sampling (Harbitz, 1986; Hohenbichler, 1988; Ibrahim and Rahman, 1991), the random variables are sampled from a different probability density, known as the sampling density. The purpose is to generate more outcomes from the region of interest, e.g. the failure set $\mathscr{F}=\{\boldsymbol{x}\colon g(\boldsymbol{x})<0\}$. Using information from FORM/SORM analyses, good sampling densities can be constructed. According to Hohenbichler (1988), the failure probability estimate $P_{F,IS}$ by importance sampling based on SORM improvement is given by

$$P_{F,IS} \cong \Phi(-\beta_{HL}) \prod_{i=1}^{N-1} [1-\kappa_i \Psi(-\beta_{HL})]^{-1/2} \times \frac{1}{N_{IS}} \sum_{j=1}^{N_{IS}} \frac{\Phi[h_Q(\mathbf{w}_j)]}{\Phi(-\beta_{HL})} \exp\left[-\frac{1}{2}\Psi(\beta_{HL}) \sum_{k=1}^{N-1} \kappa_k w_{k,j}^2 \right] \quad (56)$$

where

$$\Psi(-\beta_{HL}) = \frac{\phi(-\beta_{HL})}{\Phi(-\beta_{HL})}, \quad (57)$$

$\boldsymbol{w}_j = \{w_{1,j}, w_{2,j},\ldots, w_{N-1,j}\}^T$ is the jth realization of an $N-1$ dimensional independent Gaussian random vector $\boldsymbol{W}$ with the mean and variance of its ith component being zero and $1/[1-\Psi+(-\beta_{HL})]$, respectively, $h_Q(\boldsymbol{w}_j)$ is the quadratic approximant in the form of rotational hyperparaboloid, and N_{IS} is the sample size for importance sampling. Further details are available elsewhere (Hohenbichler, 1988).

5. Numerical applications

5.1. Description of the problem

Consider a surface-cracked side riser pipe made of Type 304 Stainless Steel from a Boiling Water Reactor (BWR) plant. The pipe has outer diameter, $D_o = 709.2$ mm (27.92 inches), wall thickness, $t = 33.77$ mm (1.33 inches) [i.e. $R_m/t = 10$], elastic modulus, $E = 182\,700$ MPa (26 500 ksi), Poisson's ratio, $\nu = 0.3$, and reference stress, $\sigma_0 = 152$ MPa (22 ksi). These parameters were treated as deterministic variables since no significant statistical variability was found from their actual measurements. The operating temperature at BWR condition was assumed to be 288°C (550°F). The random parameters included crack size parameters, θ/π and a_0/t, yield strength, σ_y, ultimate strength, σ_u, Ramberg-Osgood constitutive parameters, α and n, and mode-I fracture toughness parameters, J_{Ic}, C, and m. The statistical properties of these variables are described below.

5.1.1. Statistical characteristics of material properties

The samples of raw data for the stress-strain and J-resistance curves of a specific pipe material (e.g. Type 304 stainless steel) at 288°C (550°F) were obtained from the following references, Landes et al. (1984), Wilkowski et al. (1985), Hiser and Callahan (1987), Van Der Sluys (1988), Schmidt et al. (1991), Chopra (1993). These data were then fitted with the Eqs. (1) and (2) to determine the constitutive model parameters α and n, and fracture toughness parameters, J_{Ic}, C, and m. (This is a standard practice in the deterministic pipe fracture evaluations.) The basic strength parameters, such as yield strength, σ_y (0.2% offset) and ultimate strength, σ_u, were determined as well. These provided the independent measurements of the random vectors $\{\sigma_y, \sigma_u\}^T$, $\{\alpha, n\}^T$, and $\{J_{Ic}, C, m\}^T$ representing pipe material properties. Following standard statistical analyses, conducted in Rahman et al. (1995), Table 1 shows the mean and covariance for each of these random vectors. It was assumed that the joint probability distribution of each vector was lognormal. This was justified via comparisons with actual data, which indicated that the marginal probability of each component of the above vectors would follow lognormal distribution reasonably well (Rahman et al., 1995). A Gaussian distribution also seems to be good choice, but there are some concerns on the possible negative realizations of some of these positive random variables, which have large coefficients of variation. Hence, $\boldsymbol{X}$ was modeled with lognormal probability although no rigorous proof is provided here to validate this assumption by comparing the multivariate joint probability distributions. Also, no correlations were permitted between the strength and toughness properties because each set of laboratory data did not always include simultaneous measurement of all properties. However, the components within each vector are correlated and their correlation characteristics are defined in the covariance matrices provided in Table 1.

The methods to generate samples of generic random vector, $\boldsymbol{X}$, which are needed in FORM/SORM and simulation analyses, require Rosenblatt transformation to obtain standard Gaussian vector, $\boldsymbol{U}$ (Rosenblatt, 1952). For special cases,

Table 1
Mean and covariance of material properties for Type 304 stainless steel pipe at 288°C (550°F)

Random vector	Mean vector	Covanance matrix
$\begin{Bmatrix} \sigma_y \\ \sigma_u \end{Bmatrix}^{a}$	$\begin{Bmatrix} 151.526 \\ 450.632 \end{Bmatrix}$	$\begin{bmatrix} 220.881 & 118.615 \\ 118.615 & 652.654 \end{bmatrix}$
$\begin{Bmatrix} \alpha \\ n \end{Bmatrix}^{b}$	$\begin{Bmatrix} 8.942 \\ 3.615 \end{Bmatrix}$	$\begin{bmatrix} 10.920 & -1.202 \\ -1.202 & 0.208 \end{bmatrix}$
$\begin{Bmatrix} J_{Ic} \\ C \\ m \end{Bmatrix}^{c}$	$\begin{Bmatrix} 1059.56 \\ 345.087 \\ 0.652 \end{Bmatrix}$	$\begin{bmatrix} 2.204\times 10^5 & -58.937 & -25.530 \\ -58.937 & 1.006\times 10^4 & 6.842 \\ -25.530 & 6.842 & 0.0242 \end{bmatrix}$

[a] Both σ_y and σ_u are in MPa unit.
[b] α and n are dimensionless; $\sigma_0 = 152$ MPa, $E = 182\,700$ MPa (see Eq. (1)).
[c] Both J_{Ic} and C are in kJ/m^2 unit with $k = 1$ mm (see Eq. (2)); m is dimensionless; Δa is to be expressed in mm unit.

when $\boldsymbol{X}$ is either correlated normal or correlated lognormal, the above transformation can be sidestepped by invoking Cholesky decomposition of the covariance matrix (Rahman et al., 1995). Further details on the sample generation of $\boldsymbol{X}$, either generic or normal/lognormal, are available in Rahman et al. (1995).

5.1.2. Statistical properties of initial flaw size

In order to perform probabilistic analysis, the probability distributions of initial crack length ratio, θ/π and crack depth ratio, a_0/t need to be specified as well. Ideally, this information should come from in-service inspection data so that any uncertainty in detected crack size can be characterized by their probability. In this example problem, it was assumed that both θ/π and a/t would follow Gaussian probability distribution with means 0.25 and 0.5, respectively, and standard deviations 0.025 and 0.05, respectively. These statistical parameters and their probability distributions were chosen quite arbitrarily only to illustrate the proposed methodology. Actual determination of their statistical properties will depend on inspection quality (e.g. measurement error in crack size), material type, and other factors. They were not considered in this study.

5.2. Probabilistic characteristics of J-integral

The second-order reliability method was applied to determine the probabilistic characteristics of J-integral for the side riser pipe for several applied moments. To understand better the relative magnitude of these moments, the Net-Section-Collapse moment ($\tilde{M}_{nsc}$) of this pipe when all random input variables are assigned to their mean values was calculated using Eq. (46), and was found to be: $\tilde{M}_{nsc} = 3.73$ MN-m. Fig. 7 shows the computed probability densities $[f_J(j_0)]$ of J-integral for values of the applied moment, $M = 1.49$ MN-m ($M/\tilde{M}_{nsc} = 0.4$), 1.87 MN-m ($M/\tilde{M}_{nsc} = 0.5$), and 2.24 MN-m ($M/\tilde{M}_{nsc} = 0.6$). They were obtained by repeated SORM analysis for various thresholds of J, i.e. by calculating the probability in Eq. (48) as a function of j_0 and then taking numerical derivative of this probability with respect to j_0. As expected, the probability mass shifts to the right when the applied moments are higher. Also presented in the same figures are the corresponding histograms of J-integral developed by conducting direct MCS for the same values of applied moments. The sample size for each Monte Carlo analysis was 10 000. The results indicate that SORM can predict probabilistic characteristics of J with very good accuracy when compared with the Monte Carlo method for all values of applied moment considered here.

Figs. 8–10 show the componential probability densities (as well as total) of J-integral computed for several values of applied moment. Three load cases were considered and they were: $M = 0.37$ MN-m ($M/\tilde{M}_{nsc} = 0.1$) in Fig. 8, $M = 0.82$ MN-m ($M/\tilde{M}_{nsc} = 0.22$) in Fig. 9, and $M = 1.49$ MN-m ($M/\tilde{M}_{nsc} = 0.4$) in Fig. 10, representing small, intermediate, and large magnitudes of applied moment, respectively. For each moment, the probability densities of J_e, J_p, and J ($J = J_e + J_p$) were computed by SORM. From Fig. 8, it ap pears that when the applied moment is small ($M = 0.37$ MN-m), the elastic component of J is

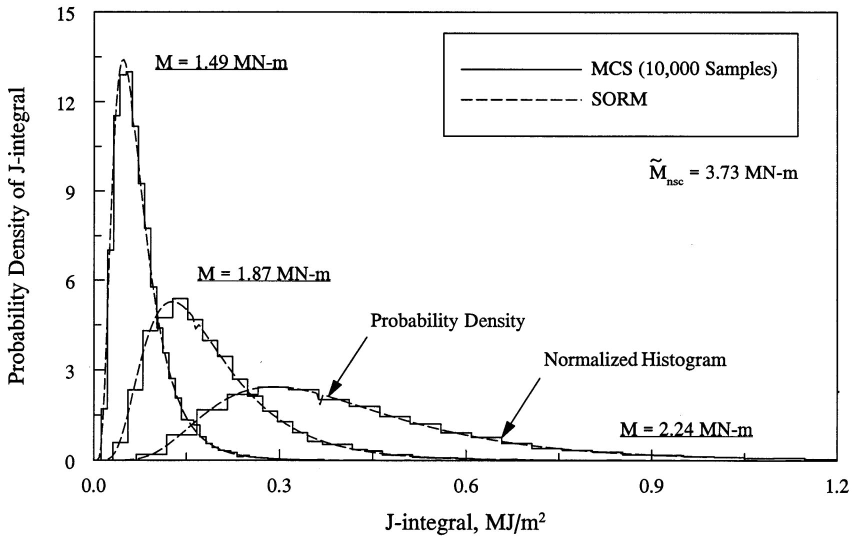

Fig. 7. Accuracy of SORM in predicting probabilistic characteristics of J-integral.

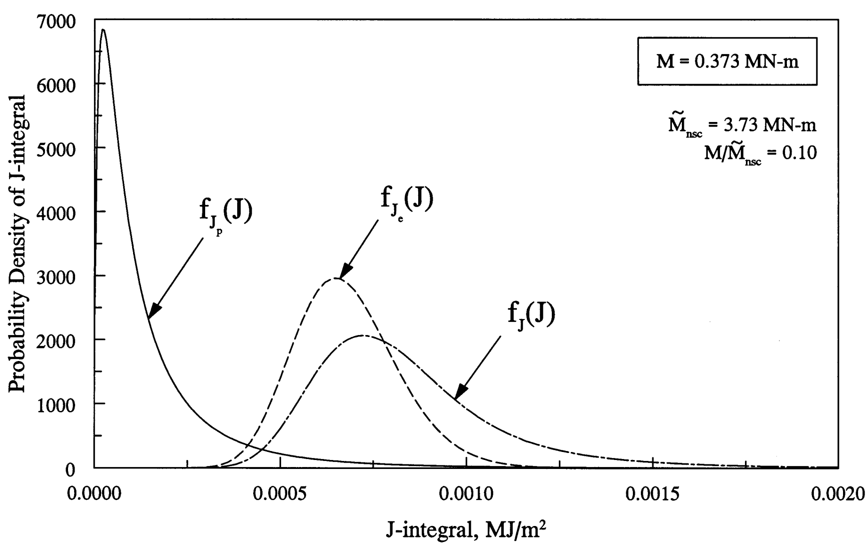

Fig. 8. Probabilistic characteristics of J_e-, J_p- and J-integrals for $M = 0.37$ MN-m.

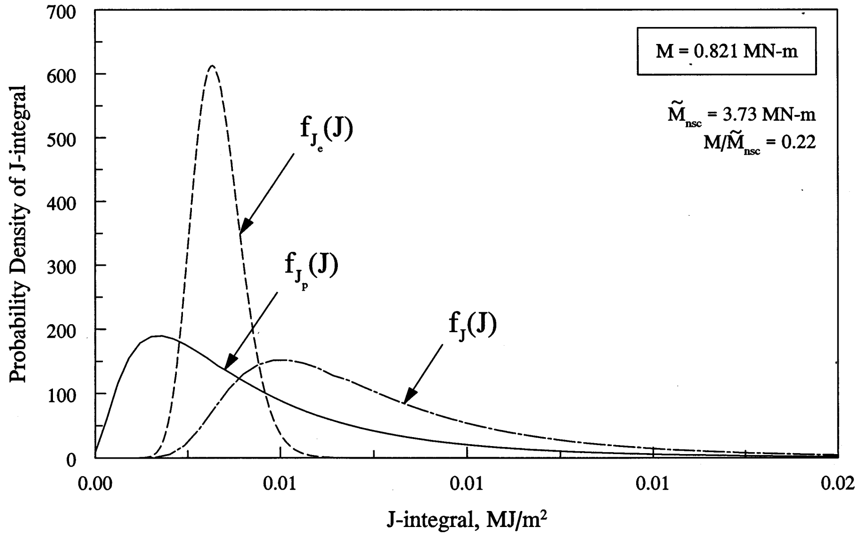

Fig. 9. Probabilistic characteristics of J_e-, J_p- and J-integrals for $M = 0.82$ MN-m.

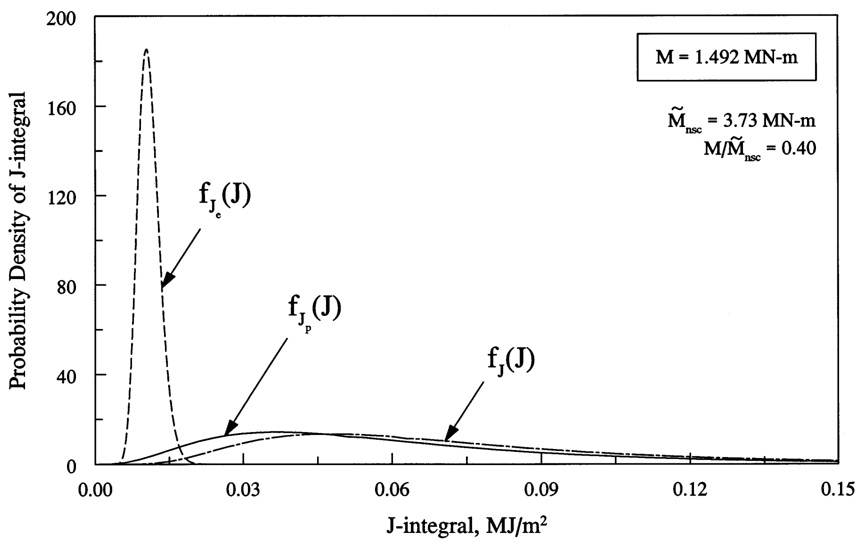

Fig. 10. Probabilistic characteristics of J_e-, J_p- and J-integrals for $M = 1.49$ MN-m.

much greater than the plastic component of J and hence, the fracture behavior is significantly governed by the elastic properties of pipe. This is equivalent to an LEFM analysis. On the other hand, when the applied load is large ($M = 1.49$ MN-m) as in Fig. 10, the plastic component of J is more pronounced and thus fully plastic fracture-mechanics analysis becomes necessary to evaluate piping integrity. Finally, in Fig. 9, when the load magnitude is somewhat intermediate ($M = 0.82$ MN-m), both components of J and hence, EPFM are needed to predict the fracture behavior of pipes.

5.3. Piping reliability assessment

Fig. 11 shows the plots of exceedance probabilities of crack initiation moment, M_i and Net-Section-Collapse moment, M_{nsc} versus applied moment M for the side riser pipe described earlier. Various reliability methods, such as FORM and SORM, and simulation methods, such as MCIS and MCS, were used to determine the failure probability. They all consistently indicate that P_F increases as M increases, and it approaches unity when M becomes very large. Compared with the failure probability due to initiation of crack growth, the failure probability based on Net-Section-Collapse was found to be much lower. Unless there is sufficient evidence that the pipe will fail with a limit-load criterion, an analysis based on the Net-Section-Collapse load (without any safety margin) may overpredict the reliability of pipes.

Fig. 11 also shows that the results obtained by the approximate methods, e.g., FORM and SORM, provide satisfactory probability estimates when compared with the results from Importance Sampling and MCS methods. No meaningful differences were found between the results of FORM and SORM and their probability estimates are virtually identical. During the performance of MCS, the sample size was varied according to the level of probability being estimated. In all cases, the sample size was targeted to be $10/\text{Min}(P_F, 1-P_F)$ for obtaining a 30% coefficient of variation of the probability estimator.

The computer program used to illustrate this example problem did not have the calculation of maximum load coded. Hence, comparisons of failure probabilities due to exceedance of maximum load and crack initiation load could not be made in this study. However, for a pipe with a surface crack, the difference between initiation and maximum loads is not typically large since the amount of crack growth is limited to the uncracked ligament of the pipe thickness. Hence, the failure probabilities of pipes based on maximum and initiation loads are expected to be very close. This situation is, however, very different for a through-wall-cracked pipe in which case a significant amount of circumferential crack growth can occur for a large-diameter pipe if the essential fracture conditions exist. In that case, probability of failure based on maximum and initiation loads can be very different. For further details, see author's previous work on through-wall-cracked pipes (Rahman, 1995; Rahman and Brust, 1995; Rahman et al., 1995; Rahman, 1996, 1997; Rahman and Kim, 1997a; Rahman and Kim 1997b; Rahman, 1998a,b; Rahman and Kim 1998).

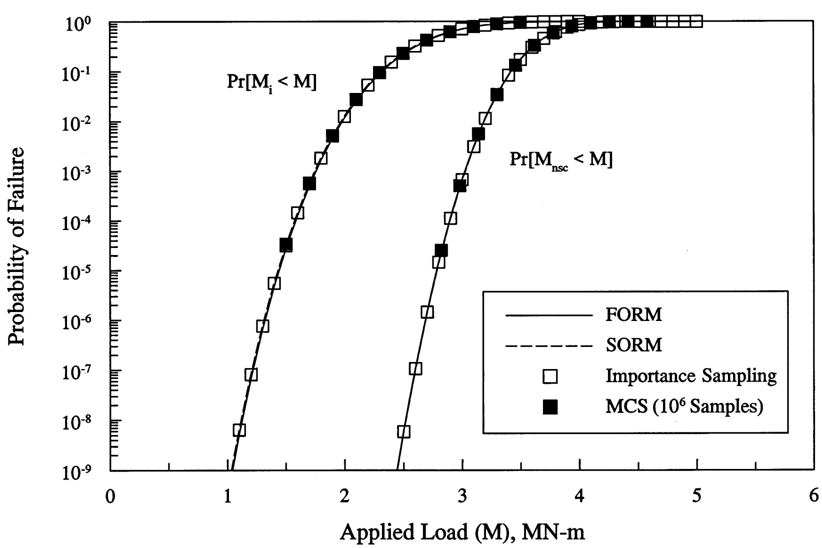

Fig. 11. Probability of failure by various reliability methods.

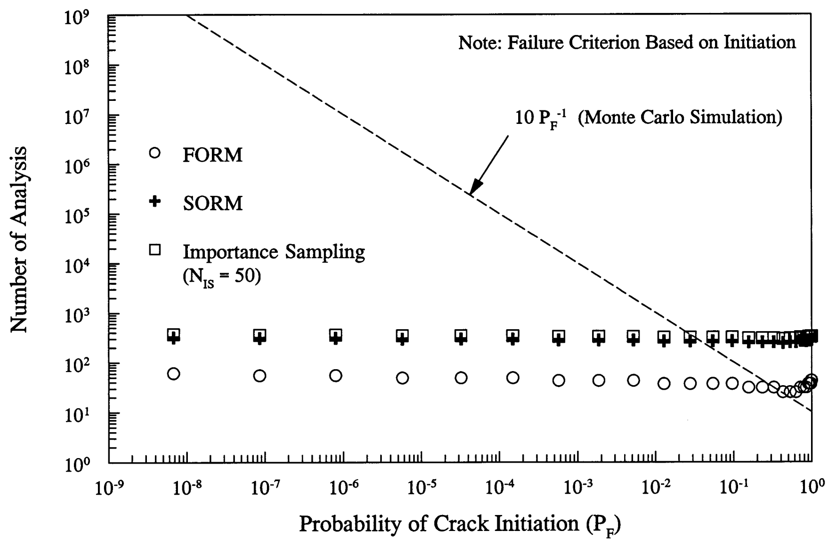

Fig. 12. Computational efficiency of FORM/SORM and MCIS.

Finally, Fig. 12 exhibits the computational effort needed to produce the results of Fig. 11 by FORM, SORM, Importance Sampling, and MCS methods. In particular, it was measured in terms of total number of deterministic analyses required by each of these methods to predict the probability of crack initiation. The plots in Fig. 12 show how the required number of analyses by these methods varies with the range of probability estimates made in this study. It appears that the number of analyses and hence, the computational effort required by FORM, SORM, and Importance Sampling methods are almost constant regardless of the probability estimates that varied from 10^{-9} to 1. On the other hand, the number of analyses required by MCS method was inversely proportional to P_F (P_F in this case is the probability of failure due to crack initiation) which suggests that for a probability estimate of about 2.5×10^{-2} or lower, a significant amount of computer time can be saved by using FORM, SORM, and Importance Sampling methods instead of using MCS. Also, the differences in number of analyses required by FORM, SORM, and Importance Sampling method are quite negligible when compared with the number of analyses required by MCS. Clearly, the FORM/SORM algorithms and Importance Sampling method are more efficient than the direct MCS and are far superior particularly when the failure probabilities are in the lower range.

6. Summary and conclusions

A probabilistic model was developed for predicting elastic-plastic fracture response of circumferentially cracked pipes with finite-length, constant-depth, internal surface flaws subject to remote bending loads. It involves engineering estimation of energy release rate, J-tearing theory for characterizing ductile fracture, and standard methods of structural reliability theory. The underlying J-estimation model is based on the deformation theory of plasticity, a constitutive law characterized by power law model for stress-strain curve, and an equivalence criterion incorporating reduced thickness analogy for simulating system compliance due to the presence of a crack. New analytical equations were developed to predict the J-integral for a surface-cracked pipe under pure bending. Both analytical and simulation methods were formulated to determine the probabilistic characteristics of J. The same methods were used later to predict the probability of crack initiation and net-section collapse as a function of the applied load. Several numerical examples are presented. The results showed that:

- The predicted J-integral by the proposed analytical methods, in particular the SC.ENG2 method, agreed reasonably well with the results of generally more accurate elastic-plastic finite element analyses.
- Current reliability methods, such as FORM and SORM, provided accurate probabilistic characteristics of J-integral and failure loads for surface-cracked pipes under bending with much less computational effort when compared with those obtained by direct MCS. Similar accuracy and computational efficiency were also demonstrated by the Importance Sampling method.
- The failure probabilities due to the exceedance of initiation load were much lower than that due to the exceedance of Net-Section-Collapse load. Large differences may exist in the results produced by each of these two failure criteria, especially when the applied load levels are smaller for which failure probabilities are also smaller. Unless there is adequate evidence that the pipe will fail with a limit-load criterion, an analysis based on Net-Section-Collapse load (without any margin) may overpredict the reliability of a piping system significantly.

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Appendix A. A modified HL-RF method

In FORM/SORM, the main effort is calculating the reliability index, $\beta_{HL} = \|\boldsymbol{u}^*\|$ by finding the design point, $\boldsymbol{u}^*$, which can be formulated as a constrained optimization problem defined by

$$\begin{aligned} &\min_{\mathbf{u}\in\Re^N} \|\boldsymbol{u}\| \\ &\text{subject to } g_U(\boldsymbol{u}) = 0 \end{aligned} \tag{58}$$

where $\Re^N$ is an N-dimensional real vector space, $\mathbf{u}\in\Re^N$ is the space of standard Gaussian vector, $\boldsymbol{U}\in\Re^N$, and $g_U(\boldsymbol{u}) : \Re^N \mapsto \Re$ is the transformed performance function in $\boldsymbol{u}$-space, and

$$\|\boldsymbol{u}\| \overset{\text{def}}{=} \sqrt{\sum_{i=i}^{N} u_i^2} \tag{59}$$

is the Euclidean $\mathscr{L}_2$-norm of the N-dimensional vector, $\boldsymbol{u}$. A modified HL-RF method, originally proposed by Hasofer and Lind (1974) and later extended by Rackwitz and Fiessler (1978) and modified by Liu and Kiureghian (1991), is one of the most widely used and robust optimization methods to solve the reliability problem in Eq. (58) (Rackwitz and Fiessler, 1978; Liu and Kiureghian, 1991). The original ML-RF method involves an iterative algorithm given by the following recursive formula

$$\boldsymbol{u}^{k+1} = \frac{1}{\|\boldsymbol{\nabla} g_U(\boldsymbol{u}^k)\|^2}[\boldsymbol{\nabla} g_U(\boldsymbol{u}^k)^T u^k - g_U(\boldsymbol{u}^k)]\boldsymbol{\nabla} g_U(\boldsymbol{u}) \tag{60}$$

where, $\boldsymbol{u}^k$ is the vector at kth iteration, $\boldsymbol{\nabla} = \{\partial/\partial u_1, \partial/\partial u_2, \ldots, \partial/\partial u_N\}^T$ is a vector of gradient operators, and $\nabla g_U(\boldsymbol{u}^k)$ is the gradient of scalar field, $g_U(\boldsymbol{u}^k)$. The algorithm proceeds iteratively until convergence is achieved, i.e. when

$$|u_i^{k+1} - u_i^k| \le \varepsilon_{con}, \text{ for all } i \tag{61}$$

and

$$|g_U(\boldsymbol{u}^*)| \cong |g_U(\boldsymbol{u}^{k+1})| \le \varepsilon_{con} \tag{62}$$

where ε_{con} is a small control parameter assigned by the user. From the past experience of authors, a value of $\varepsilon_{con} = 10^{-4}$ to 10^{-3} usually yields satisfactory estimates of β_{HL}.

To improve the robustness of Eq. (60), Liu and Kiureghian proposed a non-negative merit function, $m(\boldsymbol{u}^k)$, which is defined as (Liu and Kiureghian, 1991)

$$m(\boldsymbol{u}^k) = \frac{1}{2}\left\|\mathbf{u}^k - \frac{\boldsymbol{\nabla} g_U(\boldsymbol{u}^k)^T u^k}{\|\boldsymbol{\nabla} g_U(\boldsymbol{u}^k)\|^2}\boldsymbol{\nabla} g_U(\boldsymbol{u}^k)\right\|^2 + \frac{1}{2}c g_U(\mathbf{u}^k)^2 \tag{63}$$

where, c is some scalar positive constant. The merit function in Eq. (63) is a convenient guide for selecting step size, since it is a function of quantities already known at the current iteration point, $\boldsymbol{u}^k$. This modification greatly improves the convergence (although not strictly guaranteed) of the original HL-RF method (Liu and Kiureghian, 1991).

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