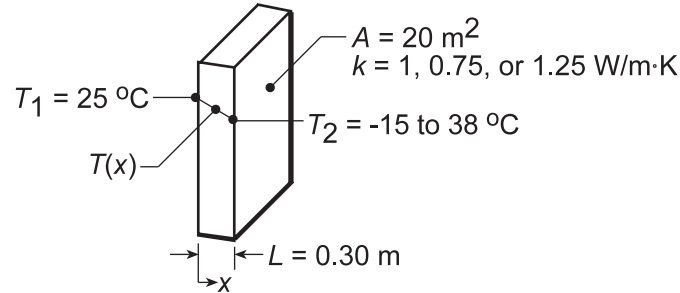


PROBLEM 1.2

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of ambient air temperatures ranging from -15 to 38°C.

SCHEMATIC:



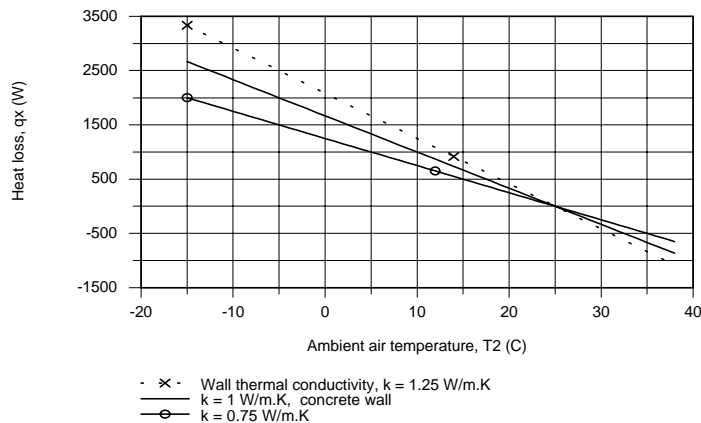
ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties, (4) Outside wall temperature is that of the ambient air.

ANALYSIS: From Fourier's law, it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear, if q''_x and k are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is $T_2 = -15^\circ\text{C}$ are

$$q''_x = -k \frac{dT}{dx} = k \frac{T_1 - T_2}{L} = 1 \text{ W/m} \cdot \text{K} \frac{25^\circ\text{C} - (-15^\circ\text{C})}{0.30 \text{ m}} = 133.3 \text{ W/m}^2. \quad (1)$$

$$q_x = q''_x \times A = 133.3 \text{ W/m}^2 \times 20 \text{ m}^2 = 2667 \text{ W}. \quad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of ambient temperature, $-15 \leq T_2 \leq 38^\circ\text{C}$, with different wall thermal conductivities, k .



For the concrete wall, $k = 1 \text{ W/m} \cdot \text{K}$, the heat loss varies linearly from +2667 W to -867 W and is zero when the inside and ambient temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

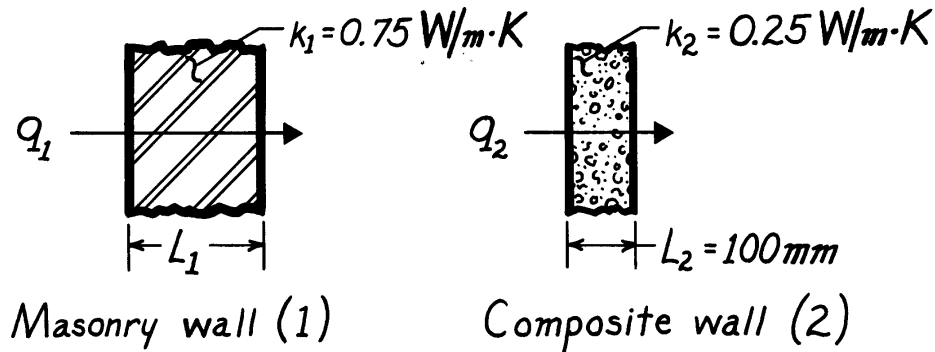
COMMENTS: Without steady-state conditions and constant k , the temperature distribution in a plane wall would not be linear.

PROBLEM 1.9

KNOWN: Masonry wall of known thermal conductivity has a heat rate which is 80% of that through a composite wall of prescribed thermal conductivity and thickness.

FIND: Thickness of masonry wall.

SCHEMATIC:



ASSUMPTIONS: (1) Both walls subjected to same surface temperatures, (2) One-dimensional conduction, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: For steady-state conditions, the conduction heat flux through a one-dimensional wall follows from Fourier's law, Eq. 1.2,

$$q'' = k \frac{\Delta T}{L}$$

where ΔT represents the difference in surface temperatures. Since ΔT is the same for both walls, it follows that

$$L_1 = L_2 \frac{k_1}{k_2} \cdot \frac{q_2''}{q_1''}$$

With the heat fluxes related as

$$q_1'' = 0.8 q_2''$$

$$L_1 = 100 \text{ mm} \frac{0.75 \text{ W/m}\cdot\text{K}}{0.25 \text{ W/m}\cdot\text{K}} \times \frac{1}{0.8} = 375 \text{ mm.} \quad <$$

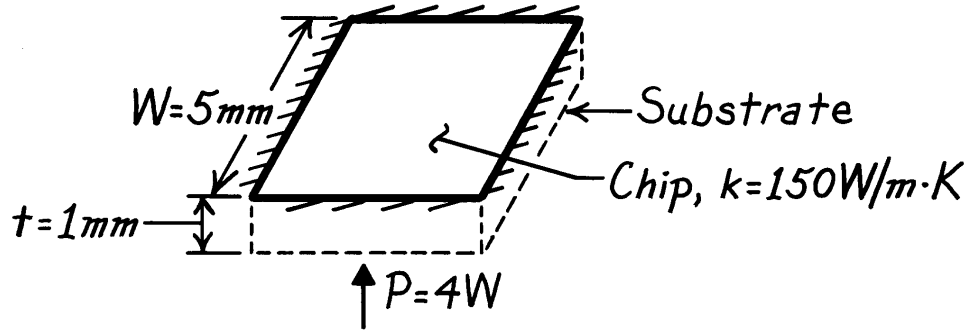
COMMENTS: Not knowing the temperature difference across the walls, we cannot find the value of the heat rate.

PROBLEM 1.11

KNOWN: Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

FIND: Temperature drop across the chip.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heat dissipation, (4) Negligible heat loss from back and sides, (5) One-dimensional conduction in chip.

ANALYSIS: All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, from Fourier's law,

$$P = q = kA \frac{\Delta T}{t}$$

or

$$\Delta T = \frac{t \cdot P}{kW^2} = \frac{0.001\text{ m} \times 4\text{ W}}{150\text{ W/m}\cdot\text{K} (0.005\text{ m})^2}$$

$$\Delta T = 1.1^\circ\text{ C.}$$

<

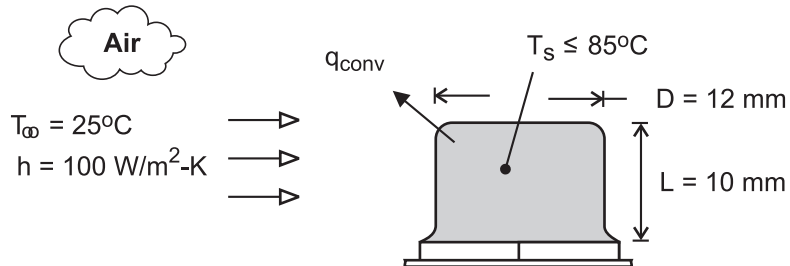
COMMENTS: For fixed P , the temperature drop across the chip decreases with increasing k and W , as well as with decreasing t .

PROBLEM 1.19

KNOWN: Length, diameter and maximum allowable surface temperature of a power transistor. Temperature and convection coefficient for air cooling.

FIND: Maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through base of transistor, (3) Negligible heat transfer by radiation from surface of transistor.

ANALYSIS: Subject to the foregoing assumptions, the power dissipated by the transistor is equivalent to the rate at which heat is transferred by convection to the air. Hence,

$$P_{\text{elec}} = q_{\text{conv}} = hA(T_s - T_\infty)$$

$$\text{where } A = \pi \left(DL + \frac{D^2}{4} \right) = \pi \left[0.012\text{m} \times 0.01\text{m} + \frac{(0.012\text{m})^2}{4} \right] = 4.90 \times 10^{-4} \text{ m}^2.$$

For a maximum allowable surface temperature of 85°C , the power is

$$P_{\text{elec}} = 100 \text{ W/m}^2 \cdot \text{K} \left(4.90 \times 10^{-4} \text{ m}^2 \right) (85 - 25)^\circ\text{C} = 2.94 \text{ W} \quad <$$

COMMENTS: (1) For the prescribed surface temperature and convection coefficient, radiation will be negligible relative to convection. However, conduction through the base could be significant, thereby permitting operation at a larger power.

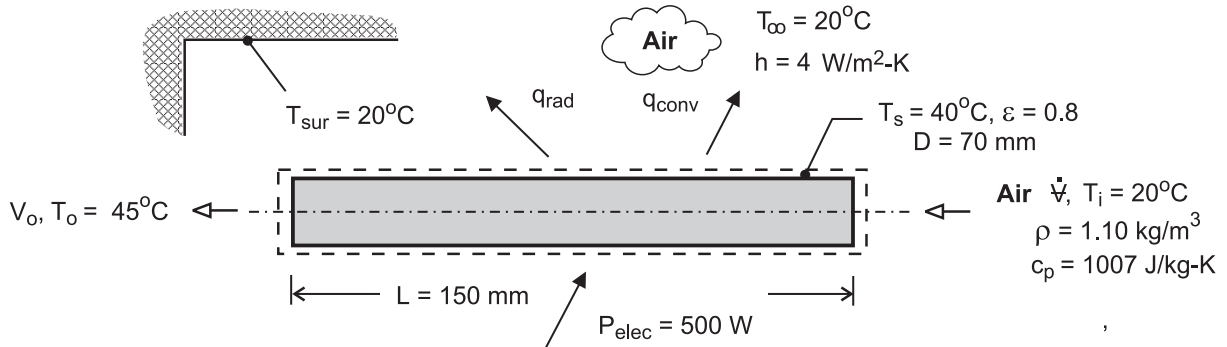
(2) The *local* convection coefficient varies over the surface, and *hot spots* could exist if there are locations at which the local value of h is substantially smaller than the prescribed average value.

PROBLEM 1.39

KNOWN: Power consumption, diameter, and inlet and discharge temperatures of a hair dryer.

FIND: (a) Volumetric flow rate and discharge velocity of heated air, (b) Heat loss from case.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible work done by fan, (5) Negligible heat transfer from casing of dryer to ambient air (Part (a)), (6) Radiation exchange between a small surface and a large enclosure (Part (b)).

ANALYSIS: (a) For a control surface about the air flow passage through the dryer, conservation of energy for an open system reduces to

$$\dot{m}(u + pv)_i - \dot{m}(u + pv)_o + q = 0$$

where $u + pv = i$ and $q = P_{elec}$. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = P_{elec}$$

$$\dot{m} = \frac{P_{elec}}{c_p(T_o - T_i)} = \frac{500 \text{ W}}{1007 \text{ J/kg} \cdot \text{K} (25^\circ\text{C})} = 0.0199 \text{ kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.0199 \text{ kg/s}}{1.10 \text{ kg/m}^3} = 0.0181 \text{ m}^3/\text{s} \quad <$$

$$V_o = \frac{\dot{V}}{A_c} = \frac{4\dot{V}}{\pi D^2} = \frac{4 \times 0.0181 \text{ m}^3/\text{s}}{\pi (0.07 \text{ m})^2} = 4.7 \text{ m/s} \quad <$$

(b) Heat transfer from the casing is by convection and radiation, and from Eq. (1.10)

$$q = hA_s(T_s - T_\infty) + \epsilon A_s \sigma (T_s^4 - T_{sur}^4)$$

Continued

PROBLEM 1.39 (Continued)

where $A_s = \pi DL = \pi(0.07 \text{ m} \times 0.15 \text{ m}) = 0.033 \text{ m}^2$. Hence,

$$q = 4 \text{ W/m}^2 \cdot \text{K} (0.033 \text{ m}^2) (20^\circ \text{C}) + 0.8 \times 0.033 \text{ m}^2 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (313^4 - 293^4) \text{K}^4$$

$$q = 2.64 \text{ W} + 3.33 \text{ W} = 5.97 \text{ W} \quad <$$

The heat loss is much less than the electrical power, and the assumption of negligible heat loss is justified.

COMMENTS: Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the dryer, causing a reduction in the density.

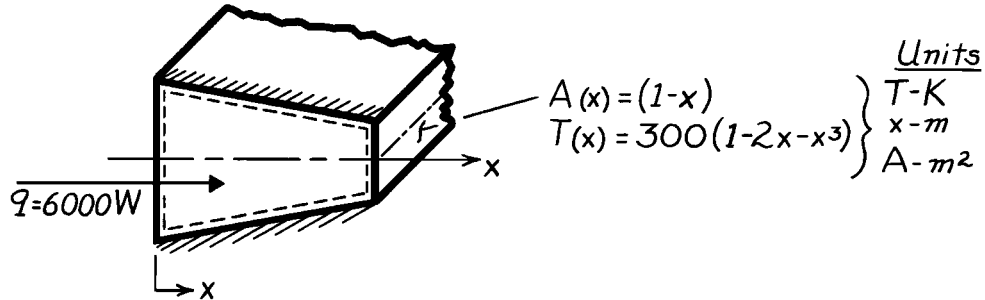
However, for the prescribed temperature rise, the change in ρ , and hence the effect on \dot{V} , is small.

PROBLEM 2.4

KNOWN: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

FIND: Expression for the thermal conductivity, k .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x -direction, (3) No internal heat generation.

ANALYSIS: Applying the energy balance, Eq. 1.11a, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

$$q_x = \text{Constant} \neq f(x).$$

Using Fourier's law, Eq. 2.1, with appropriate expressions for A_x and T , yields

$$q_x = -k A_x \frac{dT}{dx}$$

$$6000 \text{ W} = -k \cdot (1-x) \text{ m}^2 \cdot \frac{d}{dx} \left[300(1-2x-x^3) \right] \frac{\text{K}}{\text{m}}.$$

Solving for k and recognizing its units are $\text{W/m}\cdot\text{K}$,

$$k = \frac{-6000}{(1-x) \left[300(-2-3x^2) \right]} = \frac{20}{(1-x)(2+3x^2)}.$$

COMMENTS: (1) At $x = 0$, $k = 10 \text{ W/m}\cdot\text{K}$ and $k \rightarrow \infty$ as $x \rightarrow 1$. (2) Recognize that the 1-D assumption is an approximation which becomes more inappropriate as the area change with x , and hence two-dimensional effects, become more pronounced.

PROBLEM 2.16

KNOWN: Different thicknesses of three materials: rock, 18 ft; wood, 15 in; and fiberglass insulation, 6 in.

FIND: The insulating quality of the materials as measured by the R-value.

PROPERTIES: *Table A-3 (300K):*

Material	Thermal conductivity, W/m·K
Limestone	2.15
Softwood	0.12
Blanket (glass, fiber 10 kg/m ³)	0.048

ANALYSIS: The R-value, a quantity commonly used in the construction industry and building technology, is defined as

$$R \equiv \frac{L(\text{in})}{k(\text{Btu} \cdot \text{in} / \text{h} \cdot \text{ft}^2 \cdot ^\circ \text{F})}$$

The R-value can be interpreted as the thermal resistance of a 1 ft² cross section of the material. Using the conversion factor for thermal conductivity between the SI and English systems, the R-values are:

Rock, Limestone, 18 ft:

$$R = \frac{18 \text{ ft} \times 12 \frac{\text{in}}{\text{ft}}}{2.15 \frac{\text{W}}{\text{m} \cdot \text{K}} \times 0.5778 \frac{\text{Btu} / \text{h} \cdot \text{ft} \cdot ^\circ \text{F}}{\text{W} / \text{m} \cdot \text{K}} \times 12 \frac{\text{in}}{\text{ft}}} = 14.5 \left(\text{Btu} / \text{h} \cdot \text{ft}^2 \cdot ^\circ \text{F} \right)^{-1}$$

Wood, Softwood, 15 in:

$$R = \frac{15 \text{ in}}{0.12 \frac{\text{W}}{\text{m} \cdot \text{K}} \times 0.5778 \frac{\text{Btu} / \text{h} \cdot \text{ft} \cdot ^\circ \text{F}}{\text{W} / \text{m} \cdot \text{K}} \times 12 \frac{\text{in}}{\text{ft}}} = 18 \left(\text{Btu} / \text{h} \cdot \text{ft}^2 \cdot ^\circ \text{F} \right)^{-1}$$

Insulation, Blanket, 6 in:

$$R = \frac{6 \text{ in}}{0.048 \frac{\text{W}}{\text{m} \cdot \text{K}} \times 0.5778 \frac{\text{Btu} / \text{h} \cdot \text{ft} \cdot ^\circ \text{F}}{\text{W} / \text{m} \cdot \text{K}} \times 12 \frac{\text{in}}{\text{ft}}} = 18 \left(\text{Btu} / \text{h} \cdot \text{ft}^2 \cdot ^\circ \text{F} \right)^{-1}$$

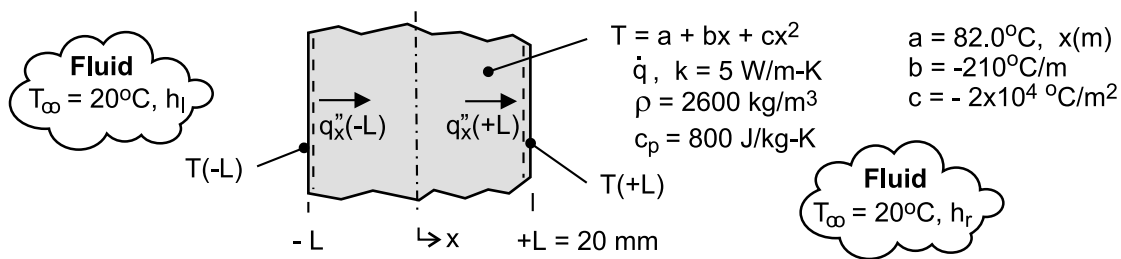
COMMENTS: The R-value of 19 given in the advertisement is reasonable.

PROBLEM 2.25

KNOWN: Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation \dot{q} while convection occurs at both of its surfaces.

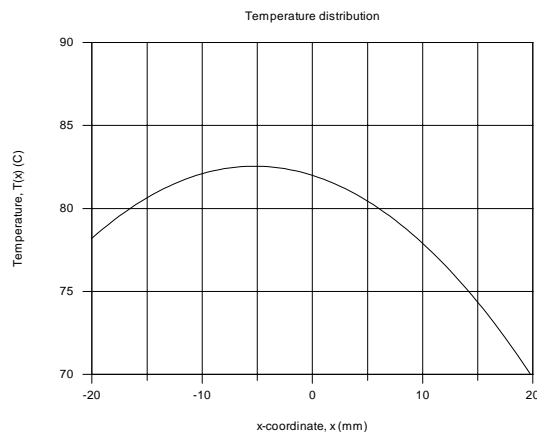
FIND: (a) Sketch the temperature distribution, $T(x)$, and identify significant physical features, (b) Determine \dot{q} , (c) Determine the surface heat fluxes, $q_x''(-L)$ and $q_x''(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces $x = L$ and $x = +L$, (e) Obtain an expression for the heat flux distribution, $q_x''(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q} = 0$), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q} = 0$; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

ANALYSIS: (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, $T(-5.25 \text{ mm}) = 83.3^\circ\text{C}$, (3) the gradient at the $x = +L$ surface is greater than at $x = -L$. Find also that $T(-L) = 78.2^\circ\text{C}$ and $T(+L) = 69.8^\circ\text{C}$ for use in part (d).



(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.15, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx} \left(\frac{dT}{dx} \right) + \frac{\dot{q}}{k} = 0 \quad \text{where} \quad T(x) = a + bx + cx^2$$

$$\frac{d}{dx} (0 + b + 2cx) + \frac{\dot{q}}{k} = (0 + 2c) + \frac{\dot{q}}{k} = 0$$

Continued

PROBLEM 2.25 (Cont.)

$$\dot{q} = -2ck = -2\left(-2 \times 10^4 \text{C/m}^2\right) 5 \text{ W/m} \cdot \text{K} = 2 \times 10^5 \text{ W/m}^3 \quad <$$

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q_x''(x) = -k \frac{dT}{dx} \quad \text{where} \quad T(x) = a + bx + cx^2$$

$$q_x''(-L) = -k[0 + b + 2cx]_{x=-L} = -[b - 2cL]k$$

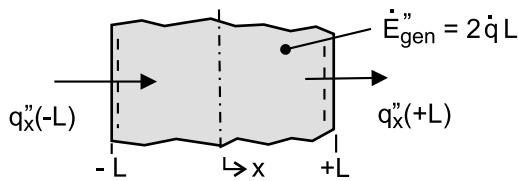
$$q_x''(-L) = -\left[-210^\circ\text{C/m} - 2\left(-2 \times 10^4 \text{C/m}^2\right) 0.020 \text{m}\right] \times 5 \text{ W/m} \cdot \text{K} = -2950 \text{ W/m}^2 \quad <$$

$$q_x''(+L) = -(b + 2cL)k = +5050 \text{ W/m}^2 \quad <$$

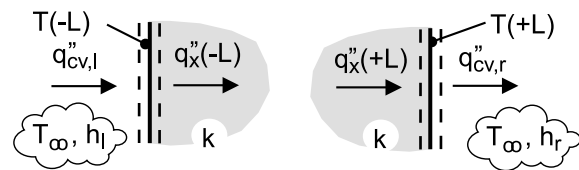
From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q_x''(-L) - q_x''(+L) + 2\dot{q}L = 0 \quad \text{or} \quad -2950 \text{ W/m}^2 - 5050 \text{ W/m}^2 + 8000 \text{ W/m}^2 = 0$$

where $2\dot{q}L = 2 \times 2 \times 10^5 \text{ W/m}^3 \times 0.020 \text{ m} = 8000 \text{ W/m}^2$, so the equality is satisfied



Part (c) Overall energy balance



Part (d) Surface energy balances

(d) The convection coefficients, h_l and h_r , for the left- and right-hand boundaries ($x = -L$ and $x = +L$, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for $T(-L)$ and $T(+L)$.

$$q_{cv,l}'' = q_x''(-L)$$

$$h_l [T_\infty - T(-L)] = h_l [20 - 78.2] \text{K} = -2950 \text{ W/m}^2 \quad h_l = 51 \text{ W/m}^2 \cdot \text{K} \quad <$$

$$q_{cv,r}'' = q_x''(+L)$$

$$h_r [T(+L) - T_\infty] = h_r [69.8 - 20] \text{K} = +5050 \text{ W/m}^2 \quad h_r = 101 \text{ W/m}^2 \cdot \text{K} \quad <$$

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q_x''(x) = -k \frac{dT}{dx} = -k[0 + b + 2cx]$$

$$q_x''(x) = -5 \text{ W/m} \cdot \text{K} \left[-210^\circ\text{C/m} + 2\left(-2 \times 10^4 \text{C/m}^2\right) x \right] = 1050 + 2 \times 10^5 x \quad <$$

Continued

PROBLEM 2.25 (Cont.)

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q_x''(x_{\max}) = 0$,

$$x_{\max} = -\frac{1050 \text{ W/m}^2}{2 \times 10^5 \text{ W/m}^3} = -5.25 \times 10^{-3} \text{ m} = -5.25 \text{ mm} \quad <$$

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q} = 0$, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) = \rho c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still $T(x) = a + bx + cx^2$. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}_{\text{st}}'' = k \frac{\partial}{\partial x} [0 + b + 2cx] = k [0 + 2c] = 5 \text{ W/m} \cdot \text{K} \times 2 \left(-2 \times 10^4 \text{ }^\circ\text{C/m}^2 \right) = -2 \times 10^5 \text{ W/m}^3 \quad <$$

(g) With no heat generation, the wall will eventually ($t \rightarrow \infty$) come to equilibrium with the fluid, $T(x, \infty) = T_\infty = 20^\circ\text{C}$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The “initial” state is that corresponding to the steady-state temperature distribution, T_i , and the “final” state has $T_f = 20^\circ\text{C}$. We’ve used T_∞ as the reference condition for the energy terms.

$$E_{\text{in}}'' - E_{\text{out}}'' = \Delta E_{\text{st}}'' = E_f'' - E_i'' \quad \text{with} \quad E_{\text{in}}'' = 0.$$

$$-E_{\text{out}}'' = \rho c_p 2L(T_f - T_\infty) - \rho c_p \int_{-L}^{+L} (T_i - T_\infty) dx$$

$$E_{\text{out}}'' = \rho c_p \int_{-L}^{+L} [a + bx + cx^2 - T_\infty] dx = \rho c_p \left[ax + bx^2/2 + cx^3/3 - T_\infty x \right]_{-L}^{+L}$$

$$E_{\text{out}}'' = \rho c_p \left[2aL + 0 + 2cx^3/3 - 2T_\infty L \right]$$

$$E_{\text{out}}'' = 2600 \text{ kg/m}^3 \times 800 \text{ J/kg} \cdot \text{K} \left[2 \times 82^\circ\text{C} \times 0.020 \text{ m} + 2 \left(-2 \times 10^4 \text{ }^\circ\text{C/m}^2 \right) \right. \\ \left. (0.020 \text{ m})^3 / 3 - 2(20^\circ\text{C})0.020 \text{ m} \right]$$

$$E_{\text{out}}'' = 4.94 \times 10^6 \text{ J/m}^2 \quad <$$

COMMENTS: (1) In part (a), note that the temperature gradient is larger at $x = +L$ than at $x = -L$. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

Continued

PROBLEM 2.25 (Cont.)

(2) In evaluating the conduction heat fluxes, $q_x''(x)$, it is important to recognize that this flux is in the positive x-direction. See how this convention is used in formulating the energy balance in part (c).

(3) It is good practice to represent energy balances with a schematic, clearly defining the system or surface, showing the CV or CS with dashed lines, and labeling the processes. Review again the features in the schematics for the energy balances of parts (c & d).

(4) Re-writing the heat diffusion equation introduced in part (b) as

$$-\frac{d}{dx}\left(-k\frac{dT}{dx}\right) + \dot{q} = 0$$

recognize that the term in parenthesis is the heat flux. From the differential equation, note that if the differential of this term is a constant (\dot{q}/k), then the term must be a linear function of the x-coordinate. This agrees with the analysis of part (e).

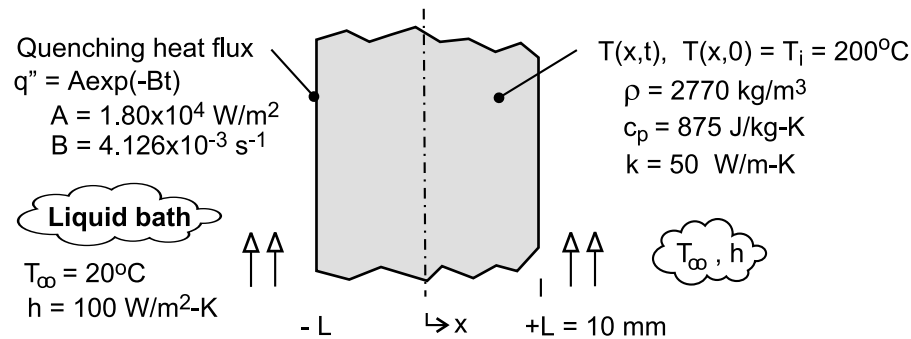
(5) In part (f), we evaluated \dot{E}_{st} , the rate of energy change stored in the wall at the instant the volumetric heat generation was deactivated. Did you notice that $\dot{E}_{st} = -2 \times 10^5 \text{ W/m}^3$ is the same value of the deactivated \dot{q} ? How do you explain this?

PROBLEM 2.45

KNOWN: Plate of thickness $2L$, initially at a uniform temperature of $T_i = 200^\circ\text{C}$, is suddenly quenched in a liquid bath of $T_\infty = 20^\circ\text{C}$ with a convection coefficient of $100 \text{ W/m}^2\cdot\text{K}$.

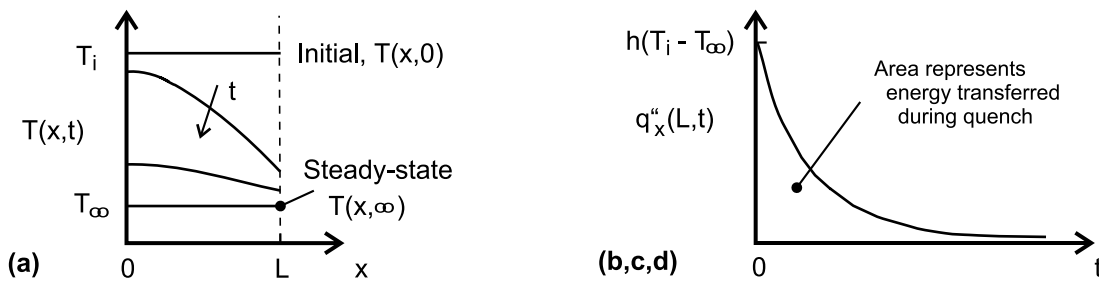
FIND: (a) On T - x coordinates, sketch the temperature distributions for the initial condition ($t \leq 0$), the steady-state condition ($t \rightarrow \infty$), and two intermediate times; (b) On $q_x'' - t$ coordinates, sketch the variation with time of the heat flux at $x = L$, (c) Determine the heat flux at $x = L$ and for $t = 0$; what is the temperature gradient for this condition; (d) By performing an energy balance on the plate, determine the amount of energy per unit surface area of the plate (J/m^2) that is transferred to the bath over the time required to reach steady-state conditions; and (e) Determine the energy transferred to the bath during the quenching process using the exponential-decay relation for the surface heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) No internal heat generation.

ANALYSIS: (a) The temperature distributions are shown in the sketch below.



(b) The heat flux at the surface $x = L$, $q_x''(L, t)$, is initially a maximum value, and decreases with increasing time as shown in the sketch above.

(c) The heat flux at the surface $x = L$ at time $t = 0$, $q_x''(L, 0)$, is equal to the convection heat flux with the surface temperature as $T(L, 0) = T_i$.

$$q_x''(L, 0) = q_{\text{conv}}''(t = 0) = h(T_i - T_\infty) = 100 \text{ W/m}^2 \cdot \text{K} (200 - 20)^\circ\text{C} = 18.0 \text{ kW/m}^2 <$$

From a surface energy balance as shown in the sketch considering the conduction and convection fluxes at the surface, the temperature gradient can be calculated.

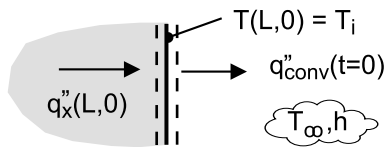
Continued

PROBLEM 2.45 (Cont.)

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0$$

$$q_x''(L, 0) - q_{\text{conv}}''(t=0) = 0 \quad \text{with} \quad q_x''(L, 0) = -k \left. \frac{\partial T}{\partial x} \right|_{x=L}$$

$$\left. \frac{\partial T}{\partial x} \right|_{L,0} = -q_{\text{conv}}''(t=0)/k = -18 \times 10^3 \text{ W/m}^2 / 50 \text{ W/m} \cdot \text{K} = -360 \text{ K/m} \quad <$$



(d) The energy transferred from the plate to the bath over the time required to reach steady-state conditions can be determined from an energy balance on a time interval basis, Eq. 1.11b. For the initial state, the plate has a uniform temperature T_i ; for the final state, the plate is at the temperature of the bath, T_∞ .

$$E_{\text{in}}'' - E_{\text{out}}'' = \Delta E_{\text{st}}'' = E_f'' - E_i'' \quad \text{with} \quad E_{\text{in}}'' = 0,$$

$$-E_{\text{out}}'' = \rho c_p (2L) [T_\infty - T_i]$$

$$E_{\text{out}}'' = -2770 \text{ kg/m}^3 \times 875 \text{ J/kg} \cdot \text{K} (2 \times 0.010 \text{ m}) [20 - 200] \text{ K} = +8.73 \times 10^6 \text{ J/m}^2 \quad <$$

(e) The energy transfer from the plate to the bath during the quenching process can be evaluated from knowledge of the surface heat flux as a function of time. The area under the curve in the $q_x''(L, t)$ vs. time plot (see schematic above) represents the energy transferred during the quench process.

$$E_{\text{out}}'' = 2 \int_{t=0}^{\infty} q_x''(L, t) dt = 2 \int_{t=0}^{\infty} A e^{-Bt} dt$$

$$E_{\text{out}}'' = 2A \left[-\frac{1}{B} e^{-Bt} \right]_0^{\infty} = 2A \left[-\frac{1}{B} (0 - 1) \right] = 2A/B$$

$$E_{\text{out}}'' = 2 \times 1.80 \times 10^4 \text{ W/m}^2 / 4.126 \times 10^{-3} \text{ s}^{-1} = 8.73 \times 10^6 \text{ J/m}^2 \quad <$$

COMMENTS: (1) Can you identify and explain the important features in the temperature distributions of part (a)?

(2) The maximum heat flux from the plate occurs at the instant the quench process begins and is equal to the convection heat flux. At this instant, the gradient in the plate at the surface is a maximum. If the gradient is too large, excessive thermal stresses could be induced and cracking could occur.

(3) In this thermodynamic analysis, we were able to determine the energy transferred during the quenching process. We cannot determine the rate at which cooling of the plate occurs without solving the heat diffusion equation.